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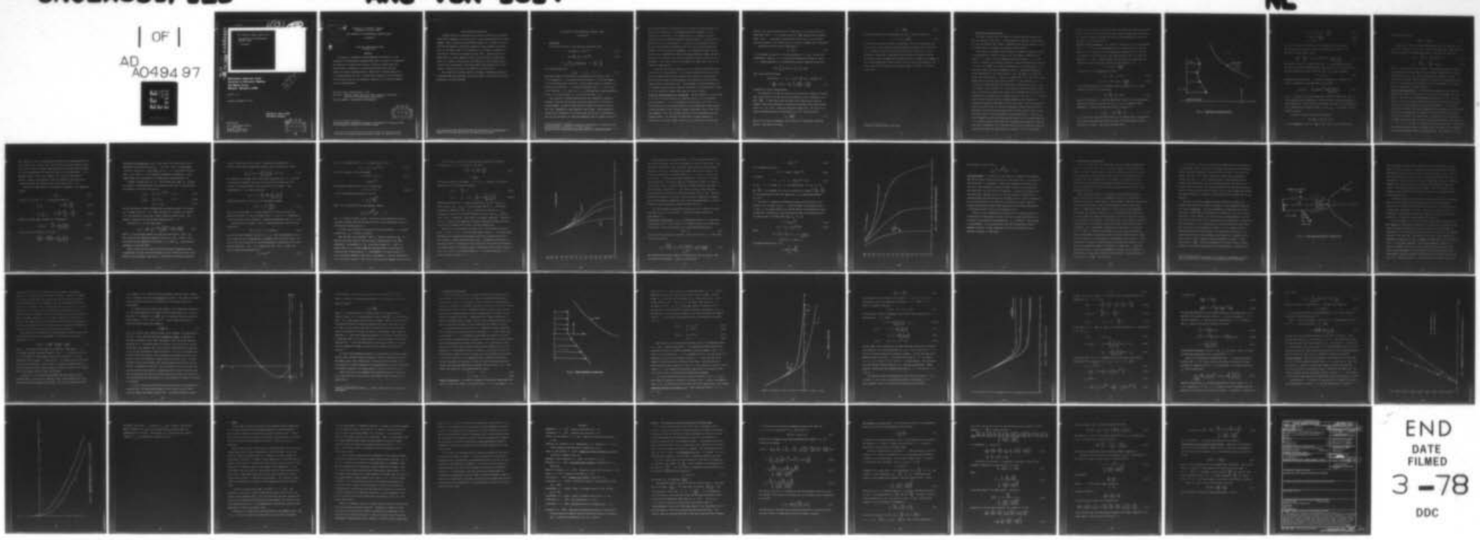

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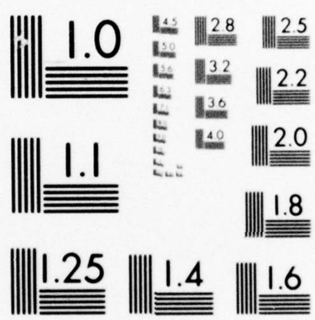

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THE QUENCHING OF TWO-DIMENSIONAL PREMIXED FLAMES

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THE QUENCHING OF TWO-DIMENSIONAL PREMIXED FLAMES

J. Buckmaster*

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ABSTRACT

An analysis is undertaken, both mathematical and numerical, of the quenching of two-dimensional premixed flames under a variety of circumstances. The discussion is divided into two parts, the first of which deals with quenching due to proximity to a surface through which there are heat losses, the second, quenching due to a shear flow of the kind experienced by a flame attached to a wire. The discussion includes a critical appraisal of flame stretch and the role sometimes claimed for it in intuitive explanations of flame quenching.

AMS (MOS) Subject Classification: 76.35
Key Words: Premixed flames, Activation energy asymptotics, Quenching, Shear flow, Two-dimensional, Flame stretch
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SIGNIFICANCE AND EXPLANATION

Premixed flames are flames that occur when the reactants are thoroughly mixed before combustion, the inner cone of a bunsen burner flame being a good example. They are distinct from diffusion flames (e.g. a candle) for which the fuel and oxygen are essentially separated, mixing together by diffusion only at the flame itself. It is possible to put a flame out (quench it) by cooling it and this can be done in several ways. Inserting a piece of metal (which is a good conductor of heat) into the flame will remove heat from it for example. Wire gauze is very effective in this respect and was used for this very purpose in the miners safety lamp developed in the 19th Century. Blowing on a flame in the right way can also cool it.

The purpose of the present paper is to examine a mathematical model of a premixed flame and investigate the precise manner in which the two cooling mechanisms can cause quenching.

THE QUENCHING OF TWO-DIMENSIONAL PREMIXED FLAMES

J. Buckmaster*

1. Introduction

The equations solved in this paper have the general form

$$Uf(y)\,\frac{\partial T}{\partial x} = \Delta T + BYe^{-\theta/T}\,, \tag{1.1a}$$

$$Uf(y)\,\frac{\partial Y}{\partial x} = \frac{1}{L}\,\Delta Y - BYe^{-\theta/T}\,, \tag{1.1b}$$

$$B \equiv \frac{\theta^2}{2(1+T_\infty)^4}\exp\left(\frac{\theta}{1+T_\infty}\right), \qquad \Delta \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

with upstream conditions

$$x \to -\infty \qquad T \to T_\infty, \quad Y \to 1\,. \tag{1.2}$$

Additional boundary conditions will be introduced in due course. Here, T is the temperature, Y the mass fraction of reactant, L the Lewis Number, θ the activation energy, and $Uf(y)$ the gas velocity which is assumed to be directed in the positive x direction and depends only on y. The scaling is such that when $L = 1$ and $f(y) \equiv 1$, a one-dimensional flame perpendicular to the x axis has a speed (U) of unity in the limit $\theta \to \infty$.

These equations, incorporating as they do the fundamental elements of diffusion, convection and reaction, have been widely used in the study of premixed flames. So much so, that they have acquired a scientific life of their own, independent in some respects of real flames. Thus although the problems to be considered in the present paper can and will be provided with some physical motivation, it should be emphasized that our primary concern is

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with an understanding of the nature of the solutions to these equations, and not with the explanation of experimentally observed phenomena. In this connection we eschew the all too common practice in combustion science of viewing the relation between mathematics and experiment as a one way street in which the only role of mathematics is to explain known experimental facts. For one thing mathematical models such as (1.1) can be used to explore the validity of intuitive physical arguments. If a mathematical model contains precisely the ingredients upon which a physical argument is based, and yet has solutions that contradict the intuitive reasoning, then the latter must be abandoned. One of the concerns of this paper is to apply such a test to the concept of flame stretch.

One of the ways in which equations (1.1) are deficient as models is that they contain no fluid mechanics. It is always conceivable that the solutions of such a system have qualitative features quite distinct from those that would be found if more accurate equations were adopted, but this possibility seems unlikely in the present case. The techniques of the present paper can be extended to equations that correctly incorporate the fluid mechanics so that it will be possible to examine this question in future studies.

Activation Energy Asymptotics - A Dichotomy. The only mathematical tool devised so far for the rational solution of equations such as (1.1) is activation energy asymptotics, i.e. the construction of solutions in the limit $\theta \to \infty$. The pioneering work in premixed flames is the one-dimensional steady analysis of Bush and Fendell (1970), and more recently Sivashinsky (1974a, 1974b, 1975) and Buckmaster (1977) have obtained results for both three-dimensional and unsteady flames. The structure of these flames is characterized by a flame-sheet of thickness $O\left(\frac{1}{\theta}\right)$ to which all the reaction is confined, and moreover, the analysis demands that the temperature in the region behind the flame sheet (where $Y \equiv 0$) differs by only an $O\left(\frac{1}{\theta}\right)$ amount from the adiabatic value $(1+T_\infty)$. This imposes strong restrictions on the class of problems that can be solved in this fashion and it is useful to examine these limitations.

Defining the location of the flame sheet by

$$x = x_f(y) \tag{1.3}$$

it is convenient to replace x by the new variable $s \equiv (x-x_f)$ so that the flame-sheet is fixed at $s = 0$, and the unburnt mixture occupies the region $s < 0$. Adding equations (1.1) eliminates the reaction terms and we may write

$$Uf(y) \int_{-\infty}^{0+} \frac{\partial}{\partial s} (T+Y)ds = \int_{-\infty}^{0+} (\Delta T + \frac{1}{L} \Delta Y)ds$$

which upon evaluation becomes

$$Uf(y)[T(s=0+) - 1 - T_\infty] - (1+x_f'^2) \frac{\partial T}{\partial s}(s=0+) + 2x_f' \frac{\partial T}{\partial y}(s=0+)$$
$$= x_f''\left[\frac{1}{L} - T(s=0+) + T_\infty\right] + \int_{-\infty}^{0+} ds \left[\frac{\partial^2 T}{\partial y^2} + \frac{1}{L}\frac{\partial^2 Y}{\partial y^2}\right] , \tag{1.4}$$

representing a global energy balance.

As noted above, in order to carry out a self-consistent asymptotic analysis, each of the terms on the left side of this equation can have a magnitude of at most $O\left(\frac{1}{\theta}\right)$, so that each of the two terms on the right must be similarly bounded. Restricting attention to situations which permit large flame deformations (i.e. $x_f' = O(1)$), there are two ways to ensure this. The first is to look for disturbances that are described on an $O(\theta)$ scale by writing

$$x_f = \theta \chi\left(\frac{y}{\theta}\right) , \tag{1.5}$$

the key to the work of Buckmaster (1977) and that of Sivaskinsky referenced earlier. The second is to write

$$L = 1 + O\left(\frac{1}{\theta}\right) \tag{1.6a}$$

and confine attention to those problems for which, consistent with this,

$$T + Y = 1 + T_{\infty} + O\left(\frac{1}{\theta}\right) , \tag{1.6b}$$

the key to the stability analysis of Sivashinsky (1977)*. At the present time a fair amount is known about solutions generated by the first choice, but their role in combustion theory is unclear since they have a propensity for local instability. Much less is known about the second type of solution, although the stability question appears more favorable (there is a band of Lewis Numbers for which the one-dimensional flame is stable for this class of disturbances), and for the most part the analysis of the present paper is confined to solutions of this type.

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*Though not clearly stated in that work.

2. Flame Near a Quenching Surface

The effect of heat losses on flames is of great practical interest and has been studied for many years. Sound mathematical results are restricted to the case of volumetric heat losses, a problem of less interest than one in which the losses occur at a boundary, the subject of this section.

The problem to be studied can be thought of as having its physical origins in the quenching of a flame at a burner rim. The flame is confined to the region $y > 0$ (Fig. 1) with the quenching surface located at $y = 0$. This surface is assumed to be chemically inactive so that the diffusion mass flux to it is zero, but $O\left(\frac{1}{\theta}\right)$ heat losses are permitted at a rate proportional to the difference between the surface temperature and T_∞, the ambient temperature of the fresh mixture. For large values of $(-x)$ the flame is far from the surface and unaffected by it. Then for a uniform flow $(f \equiv 1)$ the flame is one-dimensional and inclined at an angle $\arcsin(U^{-1})$ to the horizontal. As x increases the flame approaches the surface and is effected in two ways. There is a geometric effect present even in the absence of heat losses, which occurs (again considering a uniform flow) because the one-dimensional structure is not compatible with an adiabatic wall condition except in the case $U = 1$. Secondly, heat losses will tend to reduce the flame temperature (the temperature immediately behind the flame sheet) and this will tend to slow the flame down. Lewis and von Elbe (1961, p. 213) in a discussion of burner rims argue that quenching occurs when the heat losses reduce the flame speed to zero, so that the flame of Fig. 1 will be horizontal at the quenching point. How obvious this is depends on ones intuitive powers of course, but it is worth noting that it is in sharp contrast with results for one-dimensional flames with volumetric heat losses (Buckmaster, 1976), for which extinction occurs when the losses are sufficient to reduce the flame speed to a fraction $e^{-1/2}$ of the adiabatic value. For this reason, because it has never been shown that there are solutions of (1.1) consistent with Lewis and von Elbe's physical description, and because the interplay between the geometrical and heat loss mechanisms has never been explored, we study this problem.

Activation Energy Asymptotics. The analysis of equations (1.1) in the asymptotic limit $\theta \to \infty$ is by now straightforward, and is a generalization of Bush and Fendell's (1970) analysis of the one-dimensional deflagration wave. Reaction is confined to a thin flame sheet (the flame sketched in Fig. 1) in which the gradients of T and Y change rapidly, and both within the flame sheet and elsewhere solutions are sought as series in inverse powers of θ, i.e.

$$C \sim C_0 + \frac{1}{\theta} C_1 + O\left(\frac{1}{\theta^2}\right) .$$

In view of (1.6) it is appropriate to write

$$\frac{1}{L} = 1 - \frac{1}{\theta} \lambda, \quad \lambda = O(1) , \tag{2.1a}$$

$$T + Y \equiv \varphi \sim (1+T_\infty) + \frac{1}{\theta} \varphi_1 + O\left(\frac{1}{\theta^2}\right) , \tag{2.1b}$$

so that adding equations (1.1) together yields an equation for φ_1 valid everywhere outside of the flame sheet,

$$Uf(y) \frac{\partial \varphi_1}{\partial x} - \Delta \varphi_1 = \lambda \Delta T_0 . \tag{2.2}$$

Behind the flame sheet Y vanishes identically (there is no other mechanism to terminate the chemical reaction) and $T_0 = 1 + T_\infty$, the adiabatic flame temperature. Ahead of the flame the reaction is frozen, i.e.

$$\underline{x < x_f} \qquad Uf(y) \frac{\partial T_0}{\partial x} = \Delta T_0 . \tag{2.3}$$

T_0 is continuous across the flame sheet but its gradient jumps, the magnitude of the jump being determined from an analysis of the flame sheet structure, which we now consider. Thus within the flame sheet,

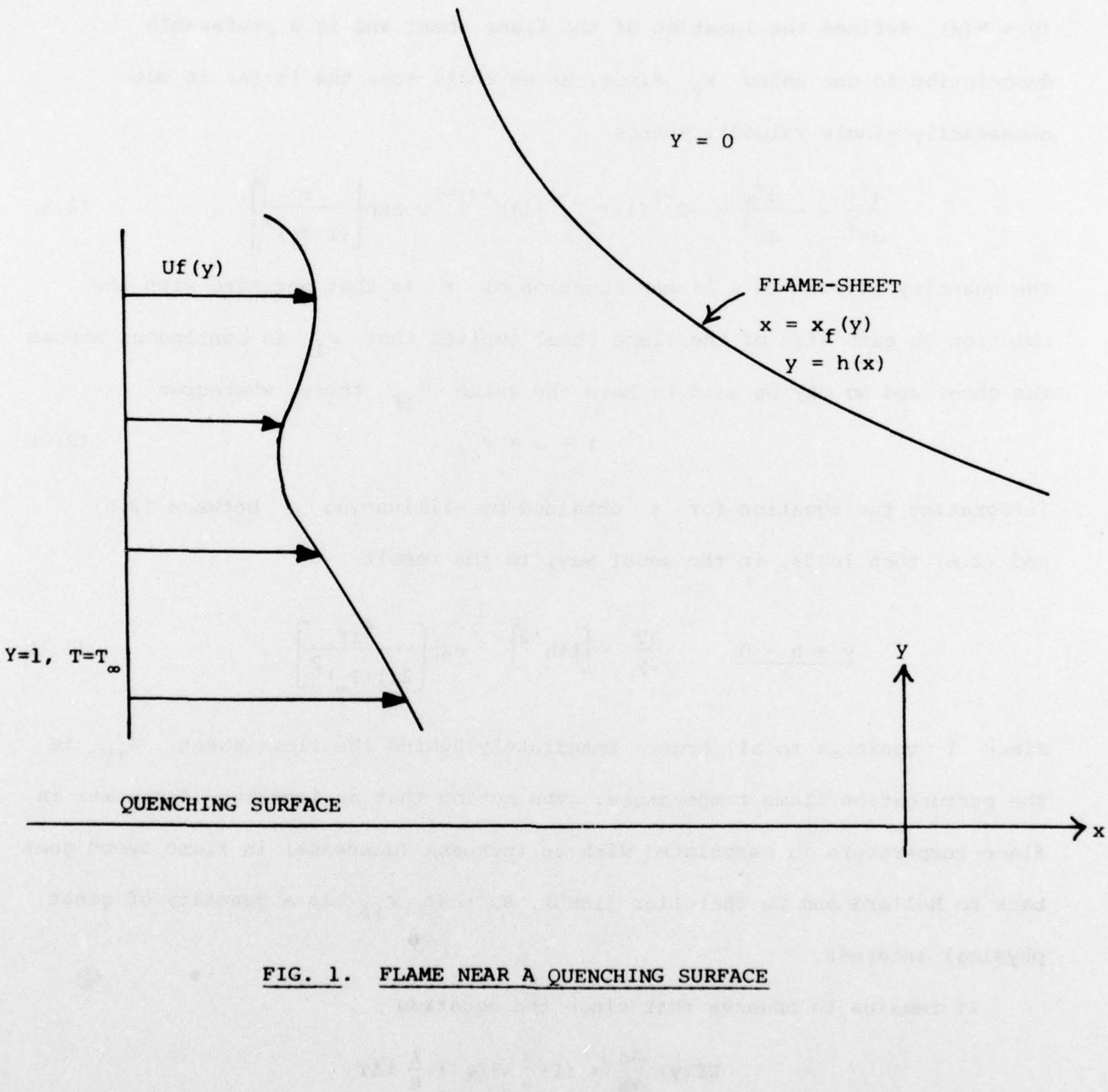

FIG. 1. FLAME NEAR A QUENCHING SURFACE

$$T \sim 1 + T_\infty + \frac{1}{\theta}\tau(\eta) + O\left(\frac{1}{\theta^2}\right) , \tag{2.4a}$$

$$Y \sim \frac{1}{\theta}\omega(\eta) + O\left(\frac{1}{\theta^2}\right) , \tag{2.4b}$$

$$y - h = \frac{1}{\theta}\eta , \tag{2.4c}$$

($y = h(x)$ defines the location of the flame sheet and is a preferable description to one using x_f since, as we shall see, the latter is not necessarily single valued), whence

$$\frac{d^2\tau}{d\eta^2} = -\frac{d^2\omega}{d\eta^2} = -2^{-1}(1+T_\infty)^{-4}[1+h'^2]^{-1}\omega \exp\left[\frac{\tau}{(1+T_\infty)^2}\right] . \tag{2.5}$$

The quantity $(\tau+\omega)$ is a linear function of η so that matching with the solution on each side of the flame sheet implies that φ_1 is continuous across the sheet and so may be said to have the value φ_{1f} there, whereupon

$$\tau + \omega = \varphi_{1f} . \tag{2.6}$$

Integrating the equation for τ obtained by eliminating ω between (2.5) and (2.6) then leads, in the usual way, to the result

$$\underline{y \to h - 0} \qquad \frac{\partial T}{\partial y} \to \left[1+h'^2\right]^{-\frac{1}{2}} \exp\left[\frac{\varphi_{1f}}{2(1+T_\infty)^2}\right] . \tag{2.7}$$

Since Y vanishes to all orders immediately behind the flame sheet, φ_{1f} is the perturbation flame temperature. The notion that an increase (decrease) in flame temperature is associated with an increase (decrease) in flame speed goes back to Mallard and Le Chatelier (1883), so that φ_{1f} is a quantity of great physical interest.

It remains to observe that since the equation

$$Uf(y)\frac{\partial\varphi}{\partial x} = \left(1-\frac{1}{\theta}\lambda\right)\Delta\varphi + \frac{1}{\theta}\lambda\Delta T$$

is valid <u>everywhere</u>, there is a jump in $\vec{\nabla}\varphi_1$ across the flame sheet whose magnitude is defined by

$$\delta(\vec{\nabla}\varphi_1) = -\lambda\delta(\vec{\nabla}T_0) \ . \tag{2.8}$$

Equations (2.2) and (2.3) must be solved subject to the conditions (2.7), (2.8), the continuity of T_0 and φ_1 across the flame sheet, and appropriate boundary conditions at the quenching surface. The latter are taken to be

$$\underline{y = 0} \qquad \frac{\partial Y}{\partial y} = 0, \quad \frac{\partial T}{\partial y} = \frac{\alpha}{\theta}\,(T - T_\infty) \tag{2.9a,b}$$

$$\text{i.e.} \qquad \frac{\partial T_0}{\partial y} = 0, \quad \frac{\partial \varphi_1}{\partial y} = \alpha(T_0 - T_\infty)\ , \tag{2.9c,d}$$

where α is a prescribed $O(1)$ constant. Such small heat losses are imposed on the problem by the requirement (1.6b), but small though they are, through their direct effect on φ_{1f} they can substantially influence the flame. In reality the heat losses through a surface will depend on the temperature field within the solid phase, but by adopting (2.9) this complication is eliminated without compromising the essential physics.

<u>The Slow Flame Approximation</u>. Activation energy asymptotics has reduced the mathematical problem to an elliptic one involving a free boundary (the flame sheet). Other than the obvious mathematical difficulties of such a problem there is a serious conceptual difficulty which arises from the fact that the solution at any point depends on the solution everywhere. It is therefore necessary to construct the solution downstream of any quenching point that might exist, where the flame sheet is terminated, and yet what form this solution must take is unknown at the present time. A resolution of this difficulty is to assume that the gas speed is much greater than the adiabatic flame speed, i.e. $U \gg 1$, $f = O(1)$, for then x derivatives are much smaller than y derivatives, the governing equations reduce to parabolic type, and these can be integrated by marching downstream towards the quenching point. The situation is in some

ways analogous to that of determining the location of the separation point in a body immersed in a viscous flow. Use of the Navier-Stokes equations, which are elliptic, presents enormous difficulties for this problem since the nature of the flow downstream of the separation point is poorly understood. If the Reynolds' Number is very large, however, the flow can be described by Prandtl's boundary layer equations, which are parabolic, and these can be integrated towards the separation point (Schlichting, 1955 p. 131).

Formally the simplification is achieved by introducing a new independent variable

$$\chi = \frac{1}{U} x \tag{2.10}$$

and then, in the limit $U \to \infty$, the equations reduce to

$$\underline{x < x_f} \qquad f(y) \frac{\partial T_0}{\partial \chi} = \frac{\partial^2 T_0}{\partial y^2}, \tag{2.11a}$$

$$\underline{|x - x_f| > 0} \qquad f(y) \frac{\partial \varphi_1}{\partial \chi} - \frac{\partial^2 \varphi_1}{\partial y^2} = \lambda \frac{\partial^2 T_0}{\partial y^2}. \tag{2.11b}$$

Moreover the flame sheet condition (2.7) simplifies to

$$\underline{y \to h - 0} \qquad \frac{\partial T_0}{\partial y} = \exp\left[\frac{\varphi_{1f}}{2(1+T_\infty)^2}\right] \tag{2.12}$$

so that (2.8) can be written in the form

$$\delta\left(\frac{\partial \varphi_1}{\partial y}\right) = -\lambda\delta\left(\frac{\partial T_0}{\partial y}\right) = \lambda \exp\left[\frac{\varphi_{1f}}{2(1+T_\infty)^2}\right]. \tag{2.13}$$

Solution for a Uniform Flow, $\lambda = 0$. This simplified problem can be solved numerically for arbitrary choices of λ and $f(y)$, but it is advantageous to restrict attention to a Lewis Number of one $(\lambda = 0)$ and velocity distributions f for which φ_1 can be explicitly expressed in quadratures. Lewis Number effects will be explored in a different but related problem in §4.

Consider a uniform flow $(f \equiv 1)$. Far from the wall, where φ_1 is small, the solution is characterized by a flat flame inclined at 45° to the horizontal. The structure in this far-field limit is

$$\underline{\chi < \chi_f} \qquad T_0 = T_\infty + e^{\chi+y-h_0}, \qquad \varphi_1 = 0 \tag{2.14a}$$

$$\underline{\chi > \chi_f} \qquad T_0 = 1 + T_\infty, \qquad \varphi_1 = 0 \tag{2.14b}$$

$$\chi_f = -y + h_0 , \tag{2.14c}$$

and this provides the initial condition for the general problem. The constant h_0 is chosen so that at $\chi = 0$, where the flame is a distance h_0 from the wall, the influence of the latter is small enough to validate (2.14). The numerical integration can then proceed downstream from $\chi = 0$.

The value of φ_1 at the flame sheet is

$$\varphi_{1_f}(\chi) = \frac{\alpha}{\pi^{1/2}} \int_{-\infty}^{\chi} ds \frac{[T_\infty - T_0(s,0)]}{(\chi-s)^{1/2}} \exp\left[-\frac{1}{4}\frac{h^2(\chi)}{(\chi-s)}\right] \tag{2.15}$$

where h is the distance between the flame sheet and the wall $(h(0) = h_0)$, and in this way the problem is reduced to one for T_0 and h alone. It is clear that the wall temperature will exceed T_∞ so that φ_{1_f} is non-positive everywhere on the flame sheet.

Useful insight into the nature of the solution can be obtained by using an approximate technique that was developed to a very sophisticated level in the study of viscous boundary layers prior to the advent of electronic calculating machines (Schlichting, 1955, p. 201). Integrating the equation for T_0 between the wall and the flame sheet yields an exact energy balance, namely

$$\frac{d}{d\chi} \int_0^h dy\ T_0 = \exp\left[\frac{\varphi_{1f}}{2(1+T_\infty)^2}\right] + h'(\chi)(1+T_\infty) \ . \tag{2.16}$$

The assumption is now made that a fairly crude approximation to T_0, provided it satisfies the boundary conditions at the wall and the flame sheet, can generate a good approximation to the left side of this equation. Thus approximating T_0 by a simple polynomial,

$$T_0 \approx 1 + T_\infty + \left(\frac{y^2}{2h} - \frac{h}{2}\right) \exp\left[\frac{\varphi_{1f}}{2(1+T_\infty)^2}\right] , \tag{2.17}$$

and substituting into (2.16) yields the approximate equation

$$\frac{2}{3} hh' \approx -1 - \frac{1}{6} \frac{h^2 \varphi'_{1f}}{(1+T_\infty)^2} \ . \tag{2.18}$$

This is not accurate when h is large because (2.17) fails to reflect the important fact that T_0 will only differ from T_∞ in the $O(1)$ neighborhood of the flame sheet, but it should be a reasonable approximation when h is $O(1)$.

When the wall is insulated $(\alpha=0)$ φ_1 vanishes identically and (2.18) has the solution

$$h^2 \approx 3(\chi_0-\chi), \quad \chi_0 = \text{constant} \tag{2.19}$$

so that the flame intersects the wall at $\chi = \chi_0$. That this phenomenon is not just a creature of the approximation is suggested by the fact that a local solution of the exact problem can be found corresponding to intersection of the wall and the flame sheet. For if it is assumed that when $(\chi_0-\chi)$ is small the sheet is described approximately by

$$h \sim C(\chi_0-\chi)^{1/2} \tag{2.20}$$

then in the neighborhood of χ_0 it is appropriate to write

$$T_0 \sim 1 + T_\infty + (\chi_0 - \chi)^{1/2} F(\zeta) \tag{2.21}$$

which is a solution of (2.11a) provided

$$F'' - \frac{1}{2}\zeta F' + \frac{1}{2}F = 0 , \tag{2.22}$$

$$\zeta = y(\chi_0 - \chi)^{-1/2} .$$

The boundary conditions (2.9c), (2.12) require that

$$F'(0) = 0, \quad F'(C) = 1 \tag{2.23a,b}$$

so that the appropriate solution of (2.22) is

$$F = A\zeta \int_B^\zeta \frac{d\zeta e^{\frac{1}{4}\zeta^2}}{\zeta^2}$$

where B is a solution of the transcendental equation

$$\frac{1}{B} = \int_0^B d\zeta \frac{(e^{\frac{1}{4}\zeta^2} - 1)}{\zeta^2} , \quad B > 0$$

and A is chosen to ensure (2.23b). As with any local downstream solution of a parabolic problem that does not make use of the initial conditions, there are undetermined constants such as C.

An exact local solution of this type can also be found when α is not zero, assuming that φ_{1_f} is locally a constant.

When the wall is not insulated ($\alpha \neq 0$), φ_1 does not vanish and φ'_{1_f} is negative since heat losses to the wall cause a steady decline in the flame temperature. The magnitude of φ'_{1_f} at any point depends on α and it is apparent that if α is large enough the right hand side of (2.18) may well become positive at some value of χ, corresponding to a local increase in h. Such an increase, assuming it does occur, corresponds to a negative flame speed in the sense that relative to the gas the flame propagates <u>towards</u> the burnt region.

These various conjectures have been tested by numerical integration. The equation for T_0 can be written in the form

$$h^2 \frac{\partial T_0}{\partial \chi} - hh'z \frac{\partial T_0}{\partial z} = \frac{\partial^2 T_0}{\partial z^2} \qquad (2.24)$$

$$z = \frac{y}{h(\chi)}$$

which must be solved in the domain $\chi > 0$, $0 \le z \le 1$ subject to the initial condition (2.14a) and boundary conditions

$$\underline{z = 0} \qquad \frac{\partial T_0}{\partial z} = 0 , \qquad (2.25a)$$

$$\underline{z = 1} \qquad T_0 = 1 + T_\infty, \quad \frac{\partial T_0}{\partial z} = h \exp\left[\frac{\varphi_{1_f}}{2(1+T_\infty)^2}\right] . \qquad (2.25b,c)$$

Backward differencing in χ yields an equation for $T_{j+1}(z) \equiv T_0(\chi_{j+1}, z)$ which contains $h_{j+1} \equiv h(\chi_{j+1})$ as well as known data at χ_j. An iterative procedure was adopted in which h_{j+1} is guessed, the equation for T_{j+1} integrated to satisfy the boundary conditions (2.25a,b), and h_{j+1} adjusted until (2.25c) is satisfied. This proved to be rapidly convergent. Integration in the z direction was undertaken by central differencing followed by inversion of the tridiagonal matrix using a Choleski decomposition.

Results for the flame shape are shown in Fig. 2 for different values of α with T_∞ fixed $(= 0.2)$. For sufficiently small values of α the flame height decreases with a monotonically decreasing slope until intersection with the wall occurs in a manner consistent with (3.10). Increasing the magnitude of α eventually leads to the appearance of an inflexion point and the curve displays a flattened portion of increasing length. Ultimately at some critical value of $\alpha(\approx 6.9)$, h has a local minimum denoted by Q, and just downstream of Q h is an increasing function of χ.

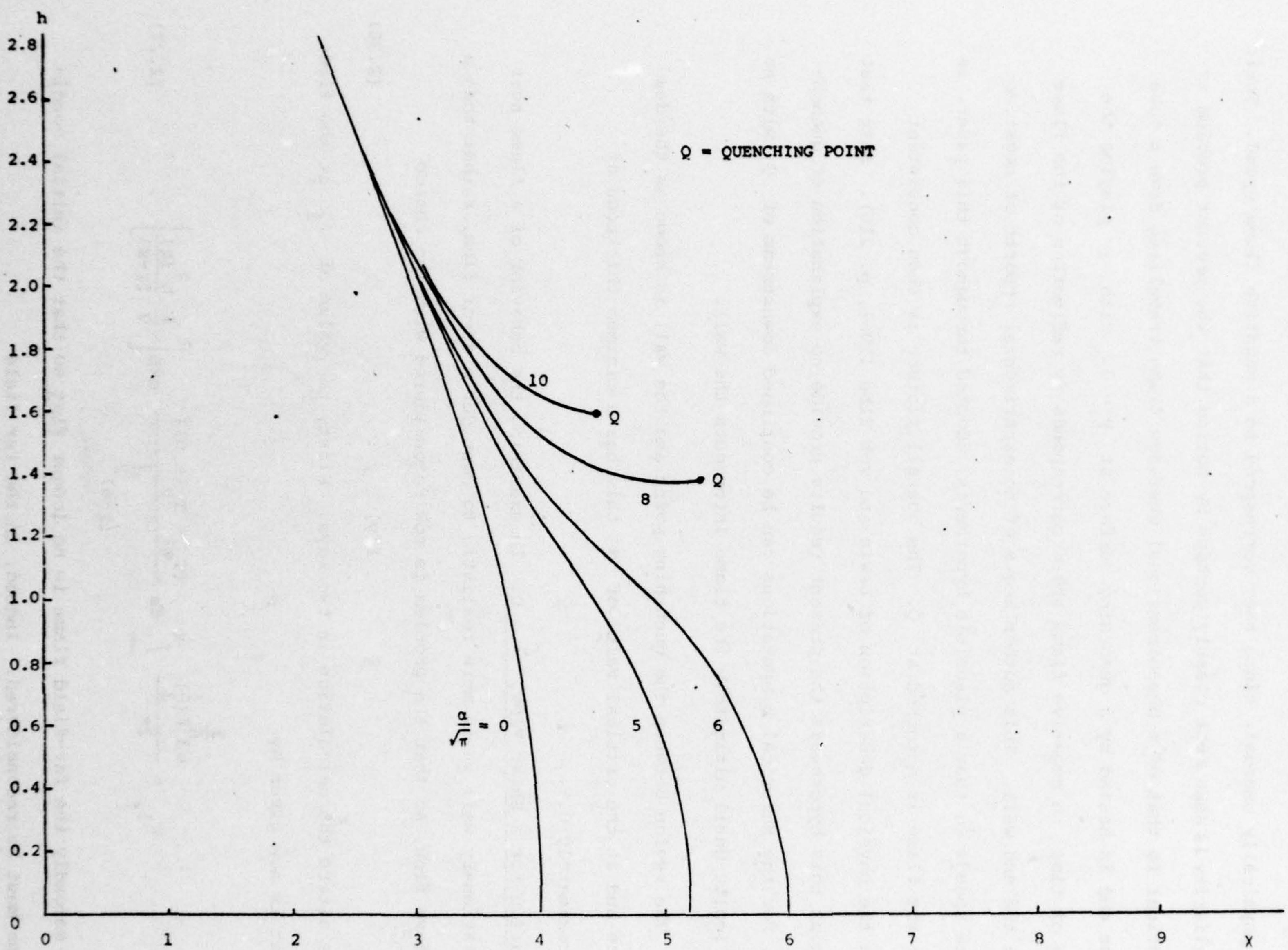

FIG. 2. EFFECT OF HEAT LOSSES FOR UNIFORM FLOW

Those portions of the curve for which h is an increasing function of χ are physically unusual, since they correspond to a negative flame speed. Their peculiarity is seen most clearly perhaps by noting that the present problem is identical to that of a one-dimensional unsteady flame travelling down a tube whose end is sealed by a quenching surface at $y = 0$, with χ playing the role of time. A negative flame speed corresponds to reflection of the flame from the end wall. This author knows of no experimental reports of negative flame speeds so that a plausible hypothesis, adopted throughout this paper, is that the flame is quenched at Q. The overall picture is then consistent with the physical description of Lewis and von Elbe (1961, p. 216). Note that without this hypothesis the present results provide no explanation of quenching, for the numerical computations can be continued downstream of Q with no difficulty until ultimately the flame intersects the wall.

The region between the quenching point and the wall is known as the dead space and at the critical value of α this has a minimum thickness of approximately 0.9.

Solution for a Shear Flow, $\lambda = 0$. In examining the behavior of a flame near a stationary wall it is more realistic to consider a shear flow, rather than a uniform flow, so that the problem is now reconsidered with the choice

$$f(y) \equiv y \ . \tag{2.26}$$

This alters the calculation in two ways. First, the value of φ_1 at the flame sheet is now given by

$$\varphi_{1_f} = \frac{\alpha 3^{\frac{1}{6}}\Gamma(\frac{1}{3})}{2\pi} \int_{-\infty}^{\chi} ds \, \frac{[T_\infty - T_0(s,0)]}{(\chi-s)^{\frac{3}{2}}} \exp\left[-\frac{1}{9}\,\frac{h^3(\chi)}{(\chi-s)}\right] \ , \tag{2.27}$$

and secondly the far-field flame is no longer flat so that the initial conditions must be reconsidered. Indeed, in the far field

$$h \sim (h_0^2 - 2\chi)^{1/2} \tag{2.28}$$

with a temperature profile

$$T_0 \sim T \;+ \exp[y - (h_0^2 - 2\chi)^{1/2}] \tag{2.29}$$

valid when

$$y \to \infty, \quad \chi \to -\infty, \quad y - (h_0^2 - 2\chi)^{1/2} = O(1) \; . \tag{2.30}$$

For as $\chi \to -\infty$ (y fixed), $T_0 \to T_\infty$ as required; when $y = h$, $T_0 = 1 + T_\infty$ and $\frac{\partial T_0}{\partial y} = 1$ as required; and (2.29) is a solution of $(h_0^2 - 2\chi)^{1/2} \frac{\partial T_0}{\partial \chi} = \frac{\partial^2 T_0}{\partial y^2}$, a valid approximation to the true equation for T_0 in the region defined by (2.30).

The results of numerical integrations starting with the initial $(\chi=0)$ profile (2.29) are shown in Fig. 3. Curves are drawn for different values of α with T_∞ once again fixed at 0.2, and the results are qualitatively similar to those for a uniform flow. The local structure near the intersection point of flame and wall is now described (when $\varphi_{1_f} = 0$) by

$$h \sim \bar{C}(\chi_0 - \chi)^{1/3} \tag{2.31a}$$

$$T_0 \sim 1 + T_\infty + (\chi_0 - \chi)^{1/3} F(\zeta) \tag{2.31b}$$

where

$$F'' - \frac{1}{3}\zeta F' + \frac{1}{3} F = 0, \qquad \zeta = \frac{y}{(\chi_0 - \chi)^{1/3}}$$

$$F'(0) = 0, \qquad F'(\bar{C}) = 1 \; .$$

The general solution for F is

$$F = \bar{A}\zeta \int_{\bar{B}}^{\zeta} d\zeta \, \frac{e^{\frac{1}{6}\zeta^2}}{\zeta^2}$$

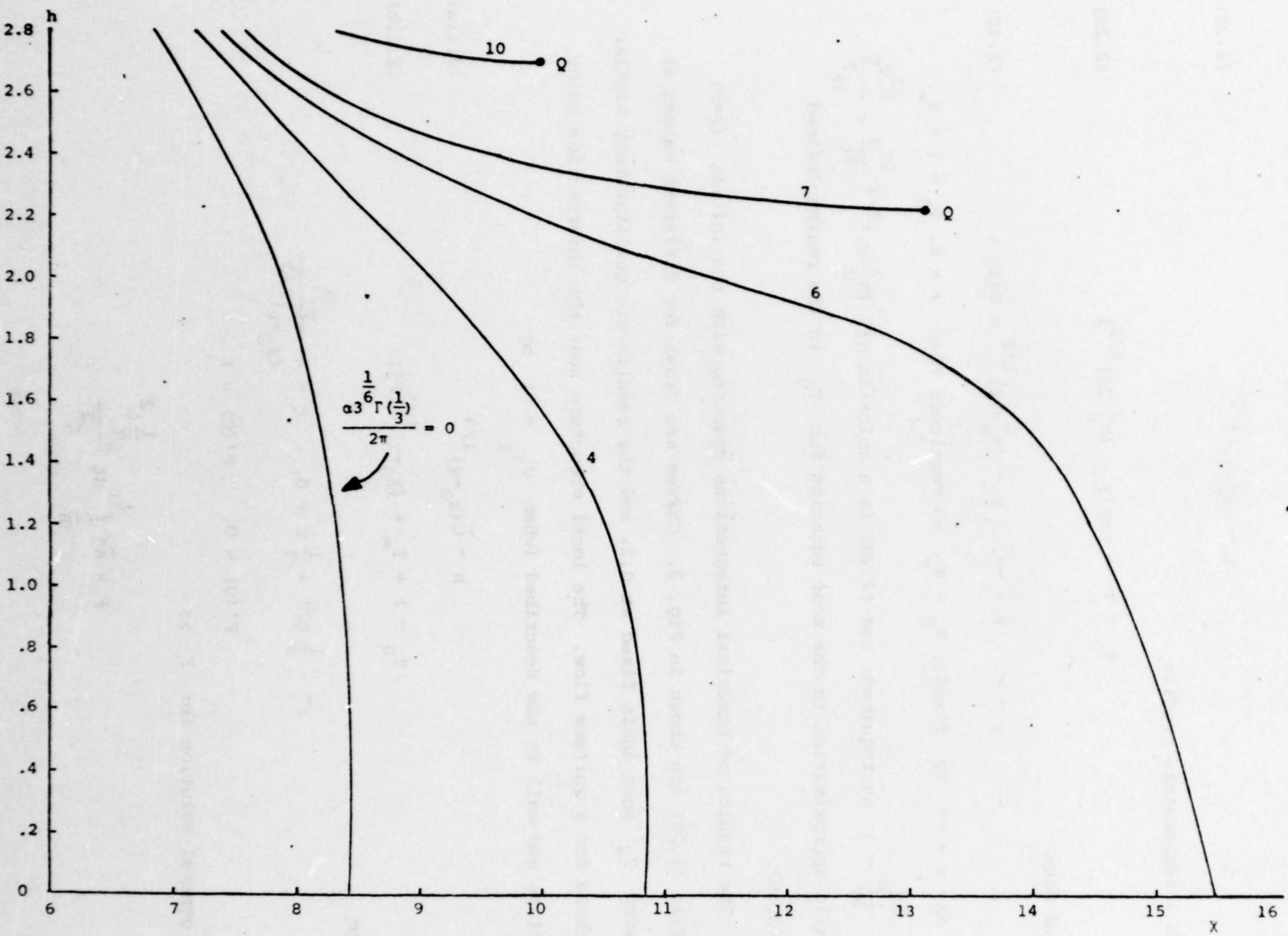

FIG. 3. EFFECT OF HEAT LOSSES FOR A SHEAR FLOW

and $\overline{B}$ must be chosen so that

$$\frac{1}{\overline{B}} = \int_0^{\overline{B}} d\zeta \, \frac{(e^{\frac{1}{6}\zeta^2} - 1)}{\zeta^2} , \quad \overline{B} > 0 .$$

Concluding Remarks. The analysis of this section is founded on the assumption that the gas speed is much greater than the flame speed so that y derivatives are much larger than x derivatives. Clearly this condition is violated in the vicinity of the intersection of wall and flame, so that it is not clear how the nonquenched solutions should be interpreted. Certainly for the shear flow one would expect flashback to occur if the heat losses are insufficient to cause quenching, and then there can be no steady solution. These considerations do not cast any doubt on the prediction of quenching of course, since the quenched solutions have bounded flame slopes and the slow flame approximation is uniformly valid all the way to the quenching point.

Finally, it can be immediately recognized that for unquenched flames in a uniform flow the flame speed increases as the wall is approached despite the fact that the perturbation flame temperature φ_{1_f} is negative. This provides a counterexample to any speculation that even for two-dimensional flames a decrease (increase) in flame temperature can always be associated with a decrease (increase) in flame speed.

3. Some Remarks on Flame Stretch

In §2 it is shown that if a flame comes close to a surface through which there are large enough heat losses, it will be quenched. The fact that the surface is a phase boundary plays no role and it may be conjectured that under some circumstances the heat transfer from one part of the combustion field to another can cause quenching even in the absence of external heat sinks. Indeed it has been argued that an appropriate measure of this heat flux is the local flame stretch, and that if this is greater than some critical value quenching will occur.

Before going any further it is appropriate to define what is meant by flame stretch and to do this in an unambiguous fashion it is necessary to define the concept of a flame surface. This is simply a sheet that characterizes the location of the flame. Thus for the problem of §2 the reaction flame sheet is a flame surface when looked at on a scale that is large compared to the preheat zone thickness divided by θ. For similar problems Lewis and von Elbe consider the flame surface defined by the locus of inflection points of the temperature distribution in the preheat zone. For problems of the type characterized by (1.5) which this author has called 'slowly varying flames' (Buckmaster, 1977), hydrodynamic disturbances are described on a scale that is $O(\theta)$ times the preheat zone thickness, and on this scale the flame is simply a hydrodynamic discontinuity of the kind discussed by Markstein (1964). This discontinuity is a flame surface. Now consider a point moving in a flame surface with a tangential velocity equal to the tangential component of the gas velocity immediately ahead of the surface. A small area Δ comprised of such points will in general be deformed as it moves over the surface, its magnitude will change and the flame will be stretched. A precise measure of this stretch is $\frac{1}{\Delta}\frac{d\Delta}{dt}$ (Williams, 1975).

Karlovitz *et al*. (1953) were the first to suggest that this stretching can, if large enough, lead to extinction of the flame, and the idea was subsequently pursued with great enthusiasm by Lewis and von Elbe (1961)*. They argue, for example, that this is the cause of blowoff of a flame stabilized behind a straight wire (Fig. 4). Such a flame experiences strong shearing from the boundary layer generated by the wire, and associated with this shearing is a positive degree of stretch. The greater the speed of the gas past the wire the greater the shear and therefore the stretch, and the claim is made that through this mechanism there is a maximum gas speed beyond which the flame will not stay attached.

Now there are several *a priori* objections that may be raised to the various claims that have been made for stretch, no less important because they are clearer *a posteriori*. The English language is an imprecise tool so that the manner in which stretch achieves what is claimed for it is not too clearly described. It will suffice to say that the basic idea expressed on p. 227 of Lewis and von Elbe is that stretch changes the rate at which heat is transferred from the reaction zone to the unburned gas, and this effects the flame speed. This can not be a local phenomenon in general, since the thermal history of fluid that travels from far upstream to the flame sheet must depend on the stretch experienced by a finite portion of the flame, if it depends upon the stretch at all. Thus a quenching criterion expressed in terms of a local stretch factor $\frac{1}{\Delta}\frac{d\Delta}{dt}$ can only be meaningful if the local stretch is representative of the stretch experienced by the flame globally. But there is no

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*That they attach great significance to the concept is suggested by the fact that it is mentioned in the Preface to the Second Edition of their book.

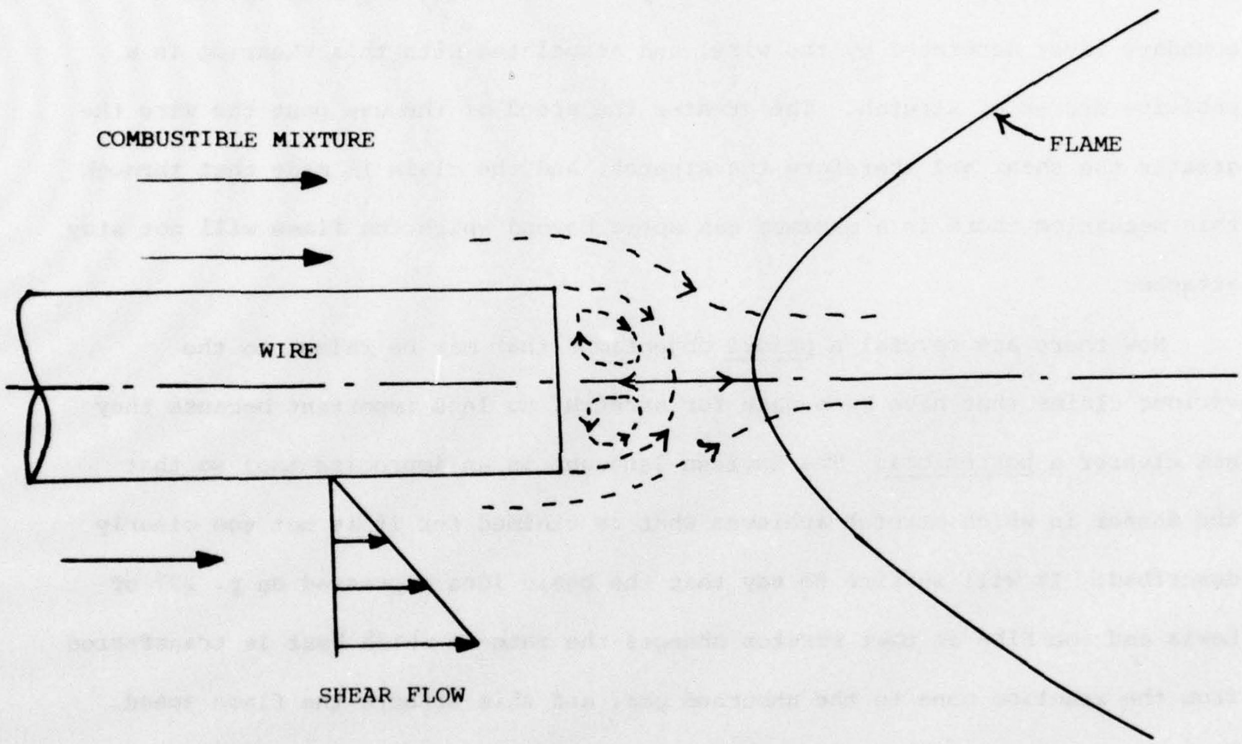

FIG. 4. FLAME STABILIZED BEHIND A STRAIGHT WIRE

way of relating the global stretch experienced by the flame of Fig. 4 at one value of the gas speed and wire diameter to that experienced under different circumstances since the flows will not be similar in general. Thus if it was true that blowoff occurs once the global stretch experienced by the flame exceeds some critical value, then a series of experiments on wires of different diameters with the gas composition fixed would inevitably yield different values of some locally defined Karlovitz Number. Only if the preheat zone thickness is much smaller than the radius of curvature of the flame will the effect be purely local.

Unfortunately there is little reason to believe that the heat transference rate is only associated with the stretch, even in some global fashion. Variations in the preheat zone thickness will also play a role and these are, in some sense, independent of stretch. One only has to think of a one-dimensional unsteady flame for which the stretch is identically zero and yet variations in thickness can occur affecting the heat flux.

Also the argument on p. 227 of Lewis and von Elbe that positive stretch must reduce the flame speed is based only on considerations of transfer of heat and active species and fails to account for the fact that the diffusion of fresh mixture is also affected by stretch but that this flux is in the opposite direction to the heat flux. The influence of positive stretch through this mechanism is apparently such as to increase the flame speed so that the overall effect is Lewis Number dependent. At a closed axisymmetric flame tip the stretch is negative and the flame speed enhanced over the adiabatic value. But at an open axisymmetric tip the stretch is still negative but here the flame speed is below the adiabatic value, something not admitted by the claim on p. 227. Curiously enough on p. 218 Lewis and von Elbe admit the role of reactant diffusion effects in explaining open tip flames, but mention it nowhere else in their extensive discussion of stretch. Thus on p. 242 where they consider blow-off from a wire they argue that the stretch is positive, the flame speed reduced as a consequence, and for large enough stretch the flame will be quenched. To be consistent with their discussion on p. 218 they would surely have to admit that for some mixture compositions blow-off is impossible, but this important test of the validity of the blow-off argument is nowhere discussed. Moreover, on p. 337 where ignition by a point source is discussed, the admission that for some mixtures stretch causes an increase in the flame speed would lead to the curious conclusion that for such mixtures an arbitrarily small ignition source will always lead to ignition.

These remarks are illuminated by a general equation governing slowly varying flames that is derived in the Appendix. This equation ((A.19) with t replaced by t') is

$$H^2 \ell n(H^2) = \beta \frac{1}{V}\frac{dV}{dt'} = \beta\left[\frac{1}{\Delta}\frac{d\Delta}{dt'} - \frac{1}{H}\frac{dH}{dt'}\right] \tag{3.1}$$

where H is the flame speed divided by the adiabatic flame speed, β is a Lewis Number dependent constant defined by (A.3b), t' is physical time, and V is the volume of a flame element defined as the product of Δ and an appropriate flame thickness δ which emerges in a natural way from the large activation energy analysis. This relation is a <u>local</u> one simply because an essential feature of slowly varying flames is that the thickness of the preheat zone is much smaller than its radius of curvature.

Equation (3.1) reveals both the thickness and Lewis Number effects discussed above. Thus it is not possible to say, without qualification, that positive stretch will always slow the flame. Nor can it be argued that when

β is negative $(L>1)$ positive stretch necessarily slows the flame, although that is certainly true when the dilatation is positive. Thus physical arguments based on stretch alone omit an essential part of the physics and so are unacceptable.

It is worth noting in passing that equation (3.1) suggests the possibility of a quenching mechanism quite different from that described in §2. Figure 5 shows how H^2 changes with $\beta \frac{1}{V} \frac{dV}{dt}$, and it is apparent that the latter has a minimum value of $(-e^{-1})$ when $H = e^{-1/2}$. Thus no flame can exist for a dilatation that satisfies the inequality

$$-\beta \frac{1}{V} \frac{dV}{dt} > \frac{1}{e}, \tag{3.2}$$

and at the critical value, when the flame is just quenched, the flame speed is not zero. Such a quenching mechanism, assuming it exists, is analogous to quenching by volumetric heat losses (Buckmaster, 1976) and not the quenching mechanism of §2. The qualification is necessary in view of the fact that the dilatation is not something that can be directly controlled but is determined in part by the instantaneous flame configuration. Thus it is conceivable that no matter what shear flows or other disturbances are applied the flame always adjusts so that the dilatation is smaller than the critical value. No flame solutions have yet been obtained that exhibit quenching of this type, and the question is presently an open one. True, Sivashinsky (1976) has proposed that a slowly varying flame located in a stagnation point flow will not exist under some circumstances, and there are obvious similarities between his analysis and the quenching mechanism proposed here, but his discussion is flawed by unresolved nonuniqueness.

So far our discussion of stretch has been restricted to broad qualitative considerations. The quantitative manner in which stretch has been used to correlate blowoff data exposes another flaw. In blowoff situations, stretch

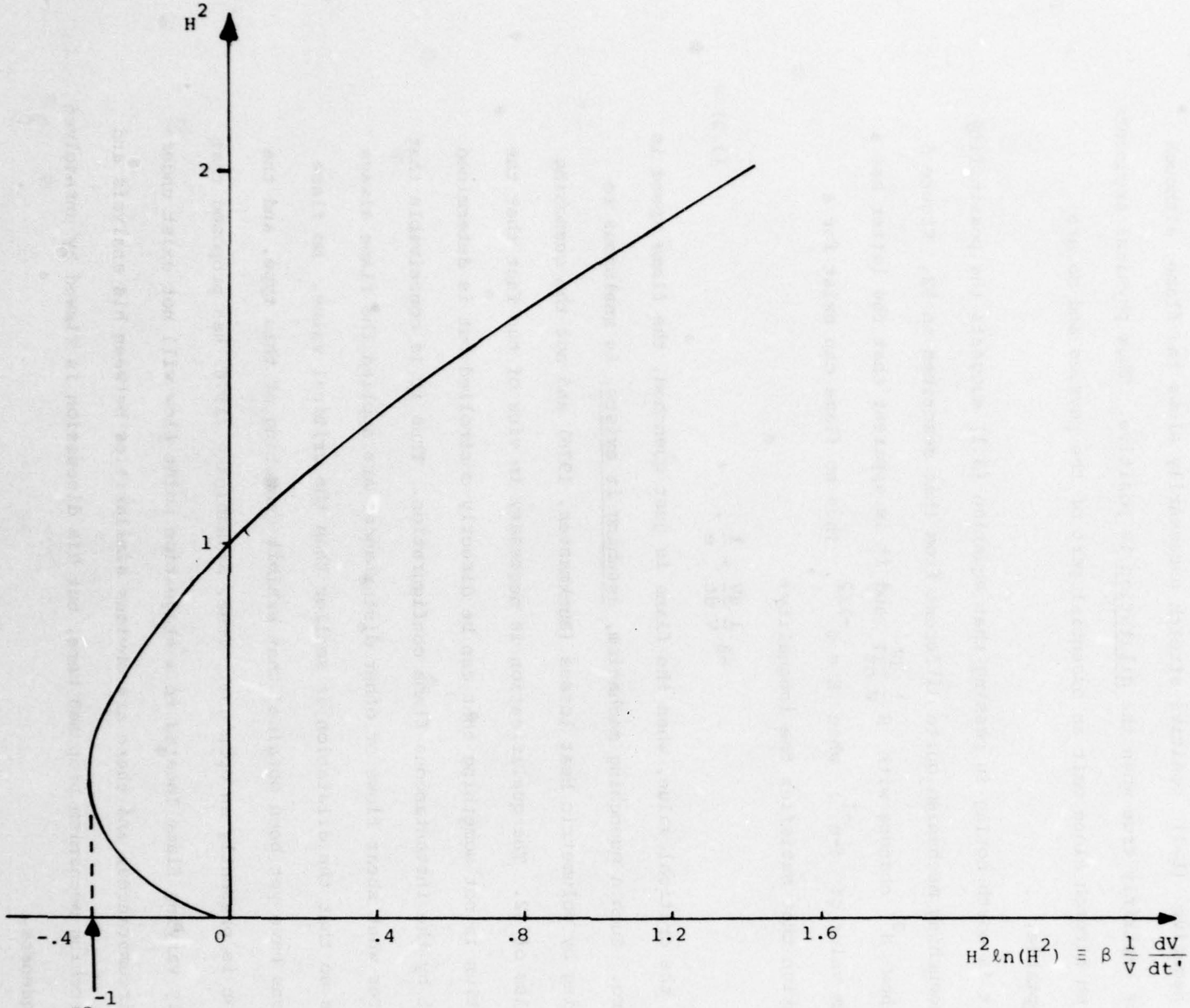

FIG. 5. VARIATION OF FLAME SPEED WITH DILATATION RATE FOR SLOWLY VARYING FLAMES

occurs because of shear and it is argued that the stretch factor $\frac{1}{\Delta}\frac{d\Delta}{dt}$ (which is difficult to determine experimentally) can be characterized by the Karlovitz Number*

$$K_a \equiv \frac{\delta'}{U'}\frac{dU'}{dy'} \tag{3.3}$$

where δ' is the thickness of the preheat zone for the one-dimensional adiabatic flame, U' is a characteristic velocity and $\frac{dU'}{dy'}$ is a characteristic shear. It is then argued (see Reed 1967 for example) that for identical burner configurations blowoff of different mixtures will occur at the same value of K_a. What is curious about this claim is that the only role the Lewis Number plays is an indirect one through its effect on the adiabatic flame speed and therefore δ', and yet one would surely expect the solutions of equations such as those discussed in §2 to depend directly on the Lewis Number through their dependence on λ. This expectation will be verified in §4 where it is shown that the response of a flame to a shear flow can depend critically on the value of λ.

In view of the preceding discussion it is this author's firm opinion that use of a local stretch criterion to explain or correlate quenching is at best misleading, and at worst simply wrong. We note that Melvin and Moss (1973), on the basis of a careful examination of experimental data, have also questioned the concept as applied to the understanding of blow-off from burner ports.

These objections leave unanswered the interesting question of whether or not a shear flow such as that of Fig. 4 can quench a flame, and this is the subject of §4.

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*However, the differences between K_a and the stretch factor are revealed in the Appendix.

4. Quenching by a Shear Flow

In this section we return to the question raised at the beginning of §3, namely whether or not distortion of a flame can generate sufficient heat transfer from one part of the combustion field to another to cause quenching. The specific problem to be examined is partly motivated by the physical situation sketched in Fig. 4 but it must be emphasized that the results of this section, despite their undoubted physical interest, probably have no relevance to the question of blowoff from a wire. Undoubtedly this is better understood in terms of the destabilizing influence of the velocity gradients behind the wire, and the stabilizing influence of heat transfer from the flame to the wire.

The model problem is shown in Fig. 6. A combustible mixture flows from left to right with a uniform velocity in the region $y > 0$ and a linear shear in the region $y < 0$. Far upstream the shear does not influence the flame which is then one-dimensional and inclined to the horizontal at an angle defined by the gas speed and the adiabatic flame speed. As x increases the flame approaches the shear region, becomes curved, and the combustion field is two-dimensional. The fundamental concern is whether or not there are values of shear and Lewis Number for which the flame will reverse and move away from the x axis with increasing x, a phenomenon that we identify with quenching. The basic equations are identical to those of §2 after the large activation energy, slow flame approximations are adopted, with the function $f(y)$ which defines the variations in gas speed chosen as follows,

$$f = 1, \qquad y > 0 \tag{4.1a}$$

$$f = 1 - Ky, \qquad y < 0 . \tag{4.1b}$$

<u>Numerical Integration</u>. The numerical problem is slightly more complicated than that of §2 since the analysis is not restricted to $\lambda = 0$. The variable y is

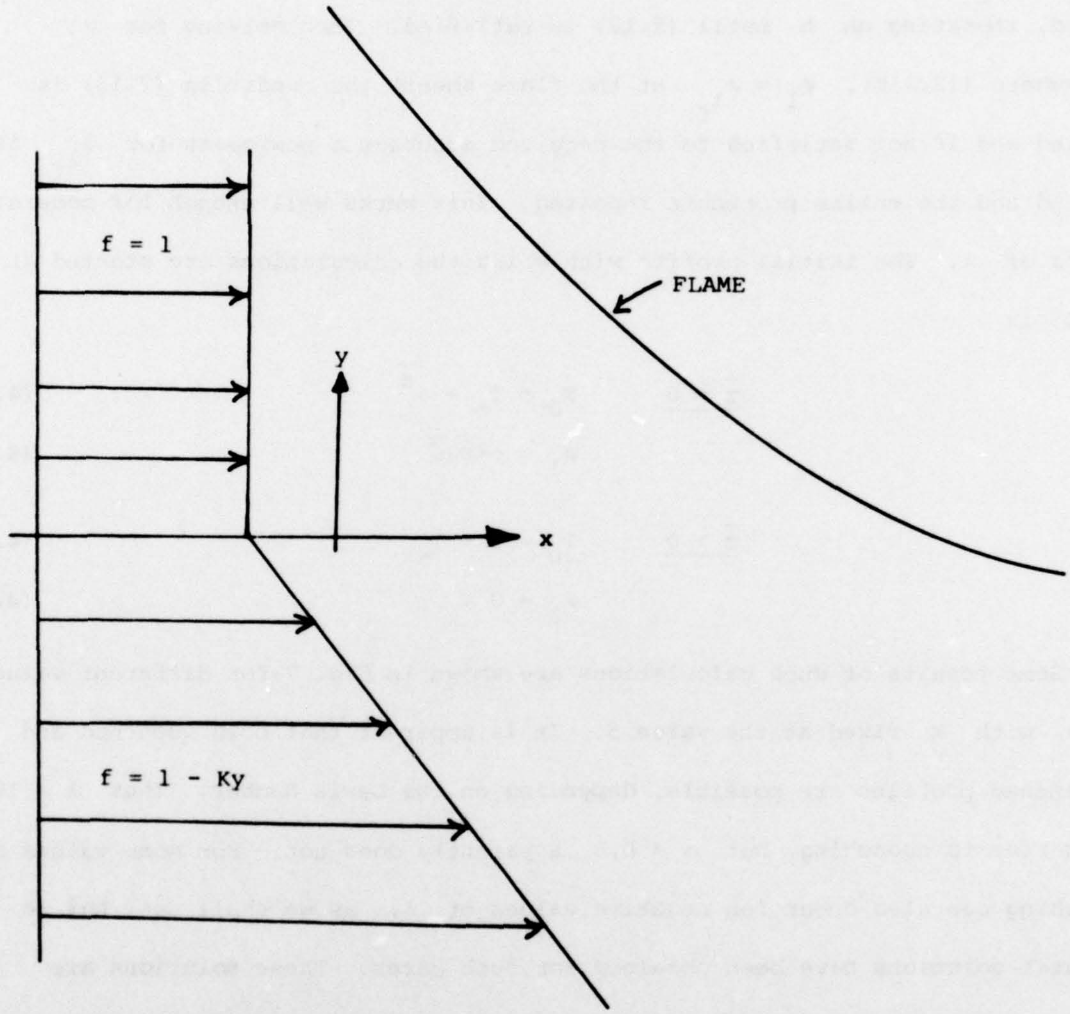

FIG. 6. FLAME APPROACHING A SHEAR FLOW

replaced by $\bar{z} \equiv y - h(x)$ in order to fix the flame sheet at $\bar{z} = 0$, and the difference equations are solved using a double iterative scheme. In this scheme φ_{1_f} is quessed and the problem for T_0 defined by (2.11a), (2.25b) solved, iterating on h until (2.12) is satisfied. Then solving for φ_1 everywhere ((2.11b), $\varphi_1 = \varphi_{1_f}$ at the flame sheet) the condition (2.13) is checked and if not satisfied to the required accuracy a new guess for φ_{1_f} is adopted and the entire procedure repeated. This works well enough for moderate values of λ. The initial profile with which the calculations are started at $\chi = 0$ is

$$\underline{\bar{z} < 0} \qquad T_0 = T_\infty + e^{\bar{z}} \tag{4.2a}$$

$$\varphi_1 = -\lambda \bar{z}\, e^{\bar{z}} \tag{4.2b}$$

$$\underline{\bar{z} > 0} \qquad T_0 = 1 + T_\infty \tag{4.2c}$$

$$\varphi_1 = 0 \,. \tag{4.2d}$$

Some results of such calculations are shown in Fig. 7 for different values of λ with K fixed at the value 5. It is apparent that both quenched and unquenched profiles are possible, depending on the Lewis Number. Thus $\lambda = 10$ gives rise to quenching, but $\lambda = 0{,}5$ apparently does not. For some values of K quenching can also occur for negative values of λ, as we shall see, but no computer solutions have been obtained for such cases. These solutions are characterized by large values of K and $(-\lambda)$, with quenching occurring at large values of χ, and convergence difficulties experienced in an attempt to find such solutions were so serious that the attempt was abandoned.

No attempt has been made to map quenching boundaries in the $\lambda - K$ parameter plane, an expensive procedure not of obvious value. Instead, the asymptotic limit $K \to \infty$ has been explored to lay bare some of the complexities of the response.

<u>Asymptotic Analysis in the Limit $K \to \infty$</u>. Limit equations valid as $K \to \infty$ in $y < 0$ are

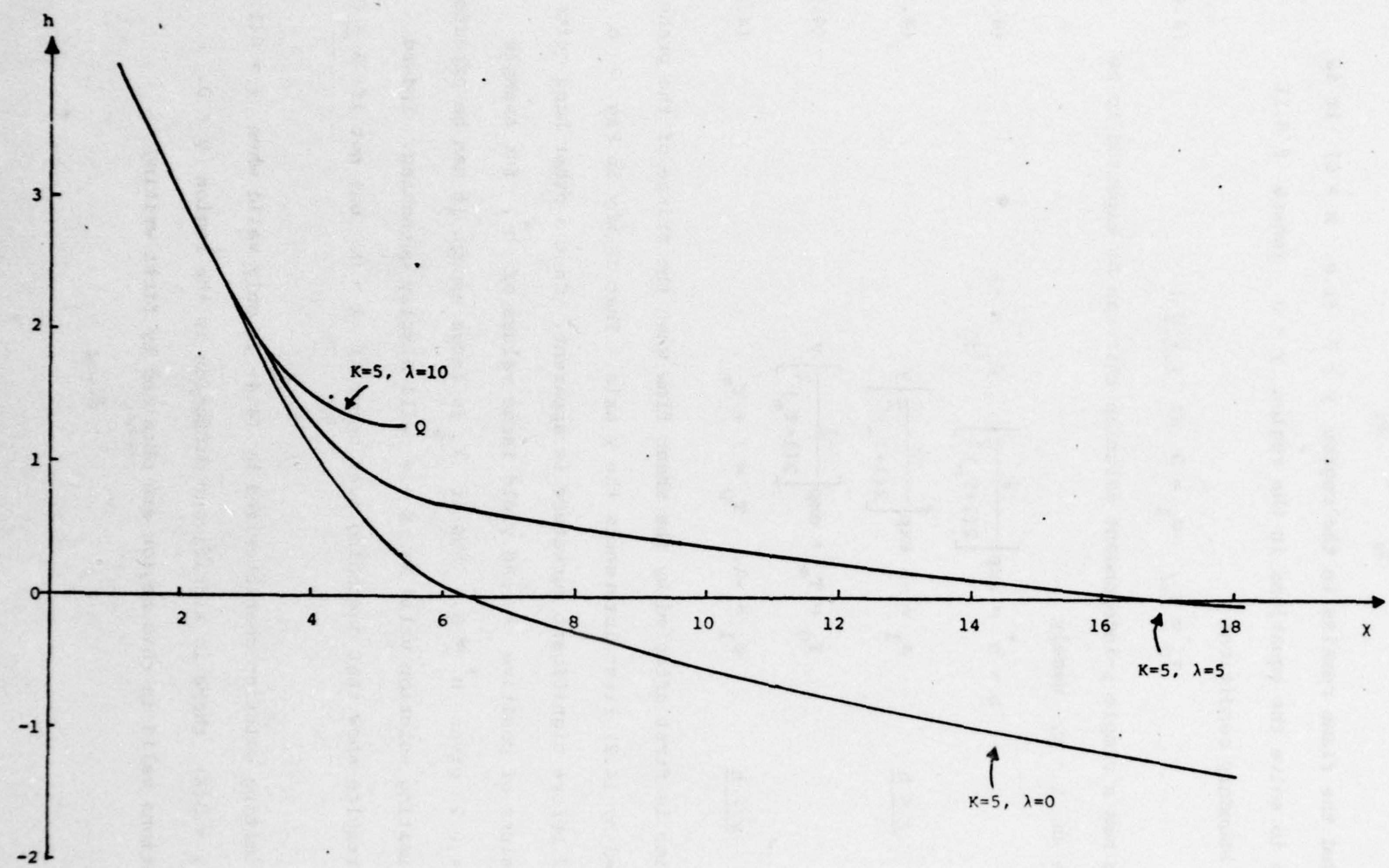

FIG. 7. FLAME IN A SHEAR FLOW

$$\frac{\partial T_0}{\partial \chi} = 0 = \frac{\partial \varphi_1}{\partial \chi} . \tag{4.3}$$

Thus provided the flame remains in the region $y > 0$ (i.e. $h > 0$) it is appropriate to solve the equations in the region $y > 0$ (where $f \equiv 1$) subject to boundary conditions

$$T_0 = T_\infty, \quad \varphi_1 = 0 \quad \text{at} \quad \underline{y = 0} . \tag{4.4}$$

This system has a simple χ-independent solution that can be expected to be appropriate as $\chi \to \infty$, namely

$$h = h^* = \exp\left[\frac{\lambda}{2(1+T_\infty)^2}\right], \quad \varphi_{1_f} = -\lambda \tag{4.5a,b}$$

$$\underline{y < h} \qquad \varphi_1 = -\lambda \exp\left[\frac{-\lambda}{2(1+T_\infty)^2}\right] y \tag{4.5c}$$

$$T_0 = T_\infty + \exp\left[\frac{-\lambda}{2(1+T_\infty)^2}\right] y \tag{4.5d}$$

$$\underline{y > h} \qquad \varphi_1 = -\lambda, \quad T_0 = 1 + T_\infty . \tag{4.5e,f}$$

Now the flame is first affected by the shear flow when the fringe of the preheat zone defined by (4.2) first intersects the χ axis. That is why in Fig. 7 h is less than 2 before significant curvature is apparent. On the other hand quite moderate values of positive λ can yield large values of h^*, for example $\lambda = 6$, $T_\infty = 0.2$ gives $h^* \approx 8$. Thus if λ is large enough it can be expected that the limiting solution valid as $K \to \infty$ will display quenching. Indeed numerical results show that quenching does occur if $\lambda > 0$, but not if $\lambda \leq 0$ (Fig. 8).

The limiting solution characterized by (4.4) is only valid when $\chi = O(1)$, for when $\chi = O(K)$ there is significant diffusion in the region $y < 0$. Limit equations valid in this region are obtained by first writing

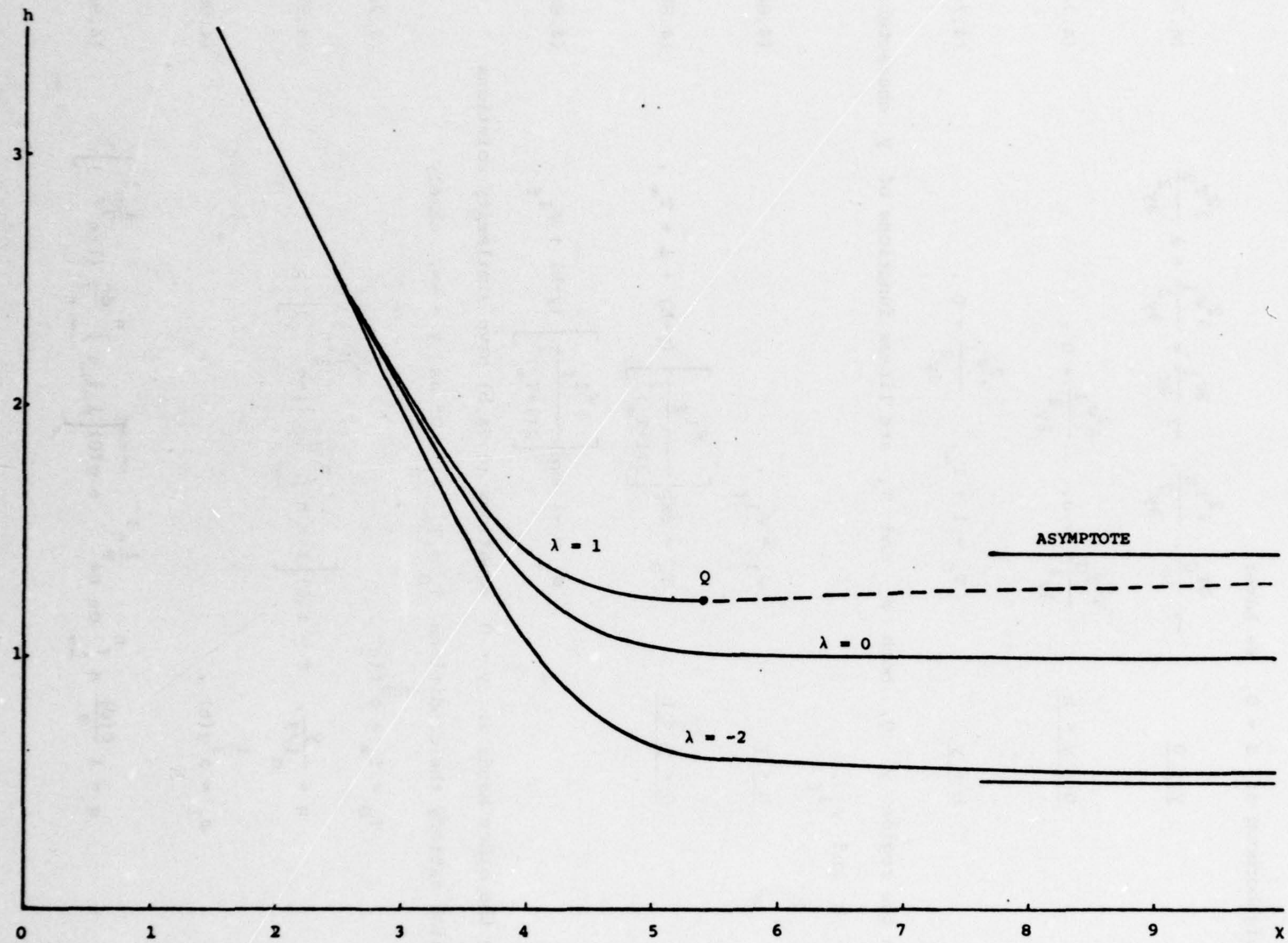

FIG. 8. FLAME SHAPE WHEN $\dot{K} \to \infty$, $\chi = O(1)$

$$\chi = \sigma K \tag{4.6}$$

so that in the limit, assuming h is positive which is certainly true in the neighborhood of $\sigma = 0$, we have:

$$\underline{y < 0} \qquad -y\frac{\partial T_0}{\partial \sigma} = \frac{\partial^2 T_0}{\partial y^2}, \quad -y\frac{\partial \varphi_1}{\partial \sigma} = \frac{\partial^2 \varphi_1}{\partial y^2} + \lambda\frac{\partial^2 T_0}{\partial y^2} \tag{4.7a,b}$$

$$\underline{0 < y < h} \qquad \frac{\partial^2 T_0}{\partial y^2} = 0, \qquad \frac{\partial^2 \varphi_1}{\partial y^2} = 0\ , \tag{4.7c,d}$$

$$\underline{h < y} \qquad T_0 = 1 + T_\infty, \qquad \frac{\partial^2 \varphi_1}{\partial y^2} = 0\ . \tag{4.7e,f}$$

In the region $y > 0$, both φ_1 and T_0 are linear functions of y characterized by h and φ_{1_f},

$$\underline{h < y} \qquad \varphi_1 = \varphi_{1_f}\ , \tag{4.8a}$$

$$\underline{0 < y < h} \qquad T_0 = \exp\left[\frac{\varphi_{1_f}}{2(1+T_\infty)^2}\right](y-h) + 1 + T_\infty\ , \tag{4.8b}$$

$$\varphi_1 = -\lambda\exp\left[\frac{\varphi_{1_f}}{2(1+T_\infty)^2}\right](y-h) + \varphi_{1_f}\ . \tag{4.8c}$$

On the other hand, in $y < 0$ equations (4.7a,b) have similarity solutions which satisfy the conditions $T_0 \to T_\infty$, $\varphi_1 \to 0$ as $y \to -\infty$, namely

$$T_0 = T_\infty + \sigma^{\frac{1}{3}} f(\eta)\ , \tag{4.9a}$$

$$\eta = \frac{y}{\sigma^{1/3}}, \quad f = f(0)\left[1 + \eta\int_{-\infty}^{\eta}\frac{d\eta}{\eta^2}\,(1-e^{\frac{1}{9}\eta^3})\right]\ , \tag{4.9b,c}$$

$$\varphi_1 = \sigma^{\frac{1}{3}} g(\eta)\ , \tag{4.9d}$$

$$g = \lambda\,\frac{f(0)}{9}\,\eta\int_{-\infty}^{\eta} d\eta\ \eta e^{\frac{1}{9}\eta^3} + g(0)\left[1 + \eta\int_{-\infty}^{\eta}\frac{d\eta}{\eta^2}\,(1-e^{\frac{1}{9}\eta^3})\right]\ . \tag{4.9e}$$

It follows that

$$\frac{f'(0)}{f(0)} = 9^{-\frac{1}{3}}\Gamma\left(\frac{2}{3}\right) , \tag{4.10a}$$

$$\frac{g'(0)}{g(0)} = -\lambda\ 9^{-\frac{5}{6}}\Gamma\left(\frac{2}{3}\right) + \frac{g(0)}{f(0)}\ 9^{-\frac{1}{3}}\Gamma\left(\frac{2}{3}\right) . \tag{4.10b}$$

The solutions obtained in the two regions must match in the sense that φ_1, T_0 and their first derivatives with respect to y are all continuous at $y = 0$. Thus (4.8) defines f, g and their first derivatives at $\eta = 0$ in terms of h and φ_{1_f} whence (4.10) may be rewritten in the form

$$\Lambda\sigma^{\frac{1}{3}} = 9^{-\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(1-h\Lambda) \tag{4.11a}$$

$$-\lambda\Lambda\sigma^{\frac{1}{3}} = -\lambda 9^{-\frac{5}{6}}\Gamma\left(\frac{2}{3}\right)(1-h\Lambda) + 9^{-\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(\varphi_{1_f} + h\lambda\Lambda) , \tag{4.11b}$$

$$\Lambda \equiv \exp\left[\frac{\varphi_{1_f}}{2(1+T_\infty)^2}\right] . \tag{4.11c}$$

These equations determine how both h and φ_{1_f} vary with σ and it is easily verified that matching with (4.5) is achieved as $\sigma \to 0$.

Since it has already been concluded that when λ is positive quenching occurs on the $\chi = O(1)$ scale, we shall only discuss the solution of these equations for negative values of λ. By eliminating h a single equation for φ_{1_f} may be written down

$$\frac{\lambda}{9^{\frac{1}{6}}\Gamma\left(\frac{2}{3}\right)}\exp\left[-\frac{\lambda}{2(1+T_\infty)^2}\right]\sigma^{\frac{1}{3}} = (\varphi_{1_f}+\lambda)\exp\left[-\frac{(\varphi_{1_f}+\lambda)}{2(1+T_\infty)^2}\right] , \tag{4.12}$$

whence it follows that φ_{1_f} decreases monotonically from its value $(-\lambda)$ at $\sigma = 0$, approaching $(-\infty)$ as $\sigma \to \infty$ (although it must be remembered that the analysis is only valid for the interval in which h is positive). Turning to h, (4.11) yields

$$-\frac{\lambda}{3}\Lambda h' = \frac{\varphi_{1_f}'}{2(1+T_\infty)^2}\left[2(1+T_\infty)^2 - \frac{2\lambda}{3} + \varphi_{1_f}\right] \tag{4.13}$$

from which it may be concluded that if λ satisfies the inequality

$$\lambda < -6(1+T_\infty)^2 \tag{4.14}$$

h is an increasing function in the neighborhood of $\sigma = 0$ so that quenching occurs with a dead space equal to h^*.

When λ lies in the interval $[-6(1+T_\infty)^2, 0]$ h decreases from its value h^* at $\sigma = 0$ and vanishes when $\sigma = \sigma_i$ where

$$\sigma_i = \frac{1}{9}\left[\Gamma\left(\frac{2}{3}\right)\right]^3 \exp\left[\frac{\lambda}{(1+T_\infty)^2}\right], \tag{4.15}$$

a result that can be checked numerically. Thus in Fig. 9 values of $\chi(h=0)$ for different values of K are shown for $\lambda = 0, -1$ and through these points are drawn straight lines of slope equal to σ_i^{-1}. For values of σ greater than σ_i the flame sheet lies in the region $y < 0$ and its progress must be followed numerically since the simple similarity solutions (4.9) are no longer appropriate. Typical results are shown in Fig. 10 and show no evidence of quenching.

Concluding Remarks. The results of numerical calculations such as those shown in Fig. 7, and the asymptotic analysis when $K \to \infty$ suggest that for some range of values of K there is a band of Lewis Numbers for which quenching does not occur, but outside this band the flame is quenched by the shear flow. The evidence suggests that the band always contains the value $\lambda = 0$. This appears to be the first sound theoretical evidence that a shear flow can quench a flame.

Quenching when λ is positive is characterized by a monotonic decrease in the flame temperature between $\chi = 0$ and the quenching point. However, if

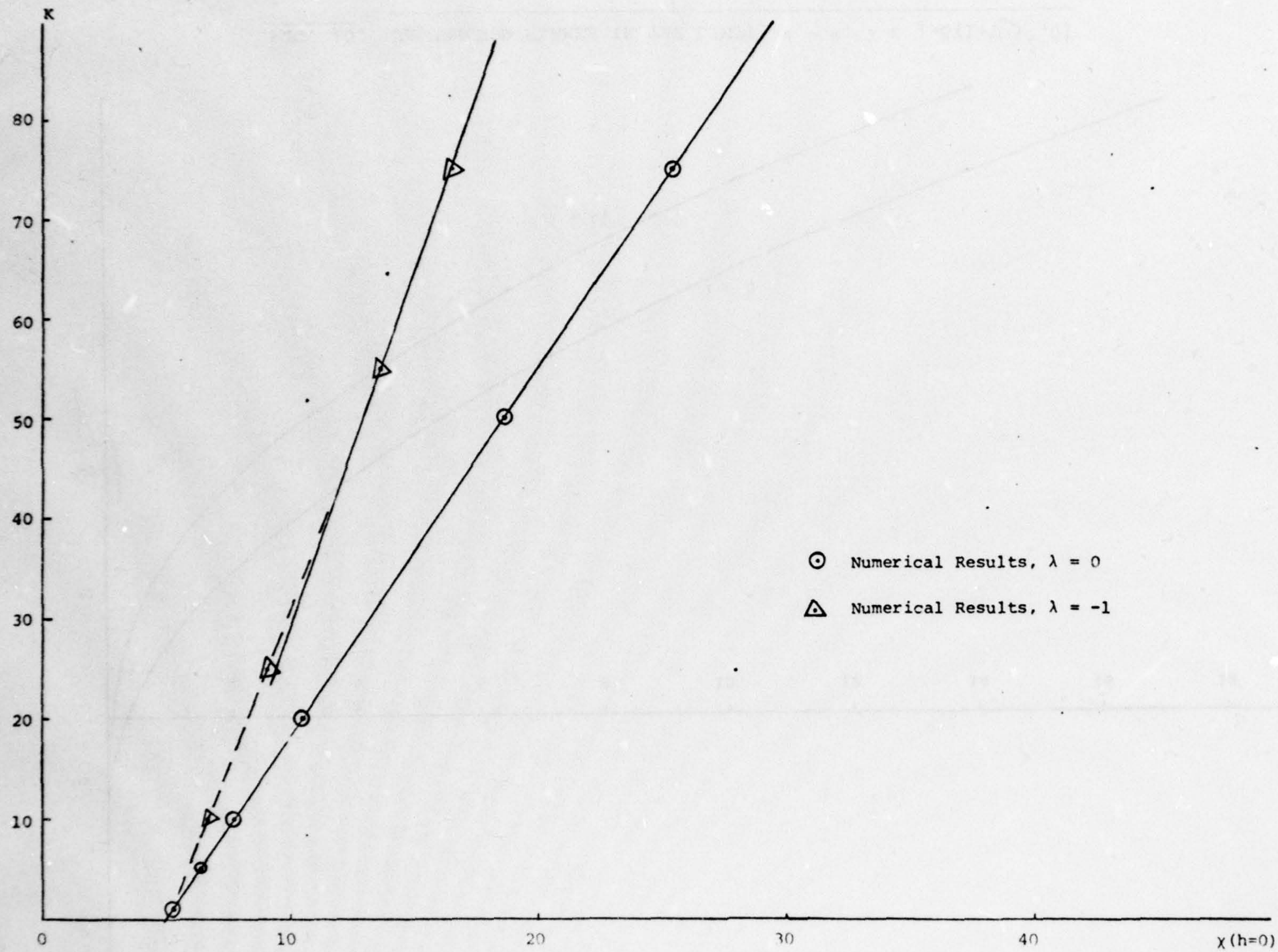

FIG. 9. CONFIRMATION OF THE ASYMPTOTIC RESULTS IN THE LIMIT $K \to \infty$

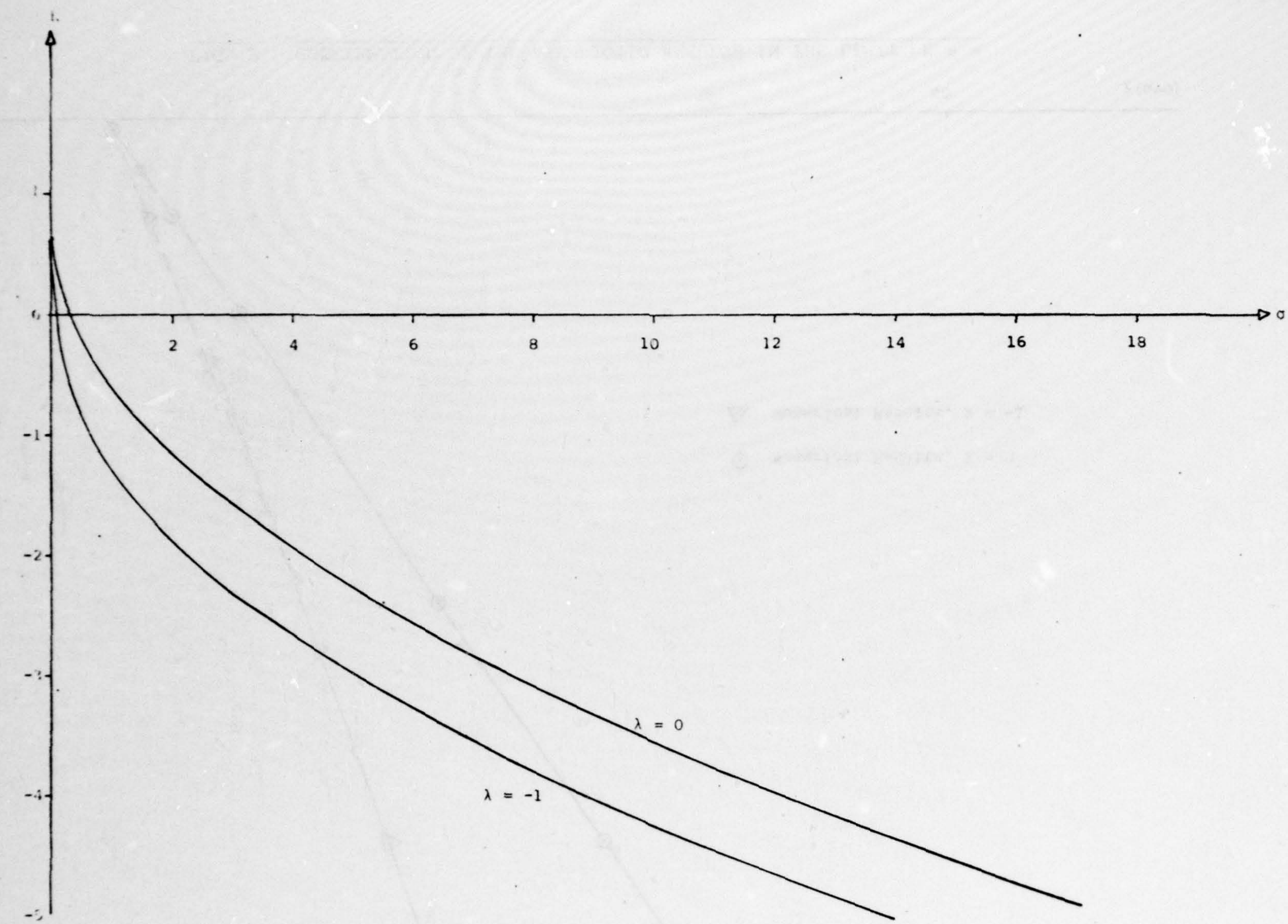

FIG. 10. UNQUENCHED FLAMES IN THE LIMIT $K \to \infty$, $\lambda \in [-6(1+T_\infty)^2, 0]$

quenching occurs when λ is negative φ_{1_f} first increases, reaching some positive maximum value, and then decreases becoming negative before the quenching point is reached. There appears to be little doubt that negative values of φ_{1_f} are necessary for quenching to occur.

5. Summary

In this paper the explicit analysis of two problems and the reinterpretation of certain results for slowly varying flames has established several facts of basic interest in the theoretical study of premixed flames. Some of these have been anticipated before but the present analysis appears to provide the first reasonably sound mathematical evidence. These results are now summarized.

When the Lewis Number is equal to one, a flame that comes close to a surface through which there are heat losses behaves in one of two ways. If the heat losses are small the dominant effect is a geometrical one which causes the flame speed to increase. No change of flame temperature is associated with this change of flame speed in the sense that when the heat losses are zero the flame temperature is everywhere equal to the adiabatic flame temperature. On the other hand if the heat losses are sufficiently large the flame speed decreases as the wall is approached, attaining the value zero at some finite distance from the wall, and this is identified with quenching. This decrease in flame speed is associated with a decrease in flame temperature below the adiabatic value.

A flame of arbitrary Lewis Number (provided $|L - 1| = O(\frac{1}{\theta})$) that approaches a shear flow can also be quenched, and whether this occurs or not depends on the parameter values. For very large shear gradients quenching does not occur for a finite band of non-positive Lewis Numbers, but outside of this band quenching does occur. In a neighborhood of the quenching point the flame temperature is below the adiabatic value.

Although all of these results are valid only for Lewis Numbers within $O(\frac{1}{\theta})$ of 1, it can not necessarily be argued that this restriction inevitably excludes large numbers of combustible mixtures. In practice activation energies are only moderately large, with values of θ between 10 and 20 being fairly typical, and for such modest numbers $O(1)$ values of λ cover the whole range of Lewis Numbers possible for a gas mixture. Thus it is conceivable that for steady premixed laminar flames the only results of activation energy asymptotics that are of practical interest are those for which $|L - 1|$ is $O(\frac{1}{\theta})$. The stability results of Buckmaster (1977) and Sivashinsky (1977) support such a view.

In spite of these remarks it would be premature at this stage to discard theoretical results such as those for slowly varying flames valid when $|1 - L|$ is $O(1)$, and one of the results of the present paper is a remarkably simple physical interpretation of the general equation governing such flames. Thus it is shown that a simple function of the flame speed is proportional to the logarithmic time derivative of a volume element of the flame, where the sign of the proportionality constant depends on the sign of $(L-1)$. The role played by changes in the flame thickness as well as the flame stretch destroys the notion that useful insight into the quenching of flames can be obtained by considerations of stretch alone; and this objection, deriving as it does from a rational analysis of the combustion equations, is quite independent of any future conclusions that may be reached about the practical utility of slowly varying flame results.

There is an important lesson to be learnt from the history of flame stretch of which the present discussion forms part. Mathematical studies are often criticized for lacking physical content, and despite the remark made in the Introduction that equations such as (1.1) have a scientific life of their own independent of known physical facts, certainly it is wise to bear in mind their physical origin. But while admitting that mathematics without physics can be sterile, it also should be noted that physics without mathematics can simply be wrong. The complexities of problems such as those examined here are too great to be unravelled by intuitive reasoning alone, so that valuable though the latter may be (the search for equation (A.19) was motivated by claims for the significance of stretch), without very strong experimental evidence or mathematical evidence the conclusions of such reasoning should not be taken too seriously.

We conclude with some observations on possible extensions of this work. The solution obtained in §2 when α is zero and there are no heat losses through the wall corresponds to a two-dimensional burner tip since the wall condition is then simply one of symmetry. It would be of great interest to study such burner tips (and their axisymmetric equivalents) for different values of λ to see whether or not open tips can be obtained under the appropriate circumstances, and such a study is currently underway. Also, as noted in the Introduction, the present techniques can be applied even when the fluid mechanics is incorporated in the proper fashion, and this facet will be explored in the future.

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Appendix. The Fundamental Equation Governing Slowly Varying Flames.

In recently published work by the present author (Buckmaster 1977) an equation is deduced that governs the general motion of a slowly varying flame. The equations that form the basis of this investigation are similar to those of the present paper but are more general in that the fluid mechanics is properly incorporated into the mathematical model. In discussing this work we shall retain the notation of the original paper rather than that of the present work in order to minimize the chances of confusion. The chief differences are that x_f, x, y, z and t are dimensional quantities, φ replaces T as the nondimensional temperature, and the flame is propagating from left to right.

A slowly varying flame is characterized by the fact that its deformation is described on the length scale $\frac{\theta\lambda}{mC_p}$ and the changes in this deformation on the time scale $\frac{\theta\rho_\infty\lambda}{m^2C_p}$. Since the analysis encompasses large deformations, with the location of the flame sheet defined by

$$x = x_f(y,z,t) \tag{A.1}$$

the variable x_f has magnitude $O\left(\frac{\theta\lambda}{mC_p}\right)$.

The asymptotic analysis must be carried out in three regions: the preheat zone of thickness $O\left(\frac{\lambda}{mC_p}\right)$; the flame sheet of thickness $O\left(\frac{1}{\theta}\frac{\lambda}{mC_p}\right)$; and the hydrodynamic regime where $|x - x_f|$ is $O\left(\frac{\theta\lambda}{mC_p}\right)$. On the hydrodynamic scale the flame is a discontinuity of the kind described by Markstein (1964) across which the temperature jumps from φ_∞ to $(1+\varphi_\infty)$. The solution on this scale is obtained in principle by solving Eulers equations of hydrodynamics on each side of the flame subject to the requirement that the mass and momentum fluxes are conserved through the discontinuity. This analysis cannot be uncoupled from that of the other regions and may be thought of as specifying the gas velocity immediately ahead of the flame (at $x = x_f + 0$) as a functional of x_f. This velocity is

$$\vec{v}(y,z,t) \equiv (v_1, v_2, v_3) \; . \tag{A.2}$$

Analysis of the combustion zone then establishes that changes in x_f are governed by the system

$$H^2 \ln(H^2) = -\beta\left[-\frac{1}{\delta_1}\frac{\partial\delta_1}{\partial t} + v_2\frac{\partial\delta_1}{\partial y} + v_3\frac{\partial\delta_1}{\partial z} + \frac{1}{\delta_1}\left(\frac{\partial^2 x_f}{\partial y^2} + \frac{\partial^2 x_f}{\partial z^2}\right) - \left(\frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}\right)\right], \tag{A.3a}$$

$$\beta = \frac{\theta\rho_\infty\lambda}{m^2 C_p}\frac{\varphi_\infty}{(1+\varphi_\infty)^2}\left[\varphi_\infty^{L-1}\int_0^{\frac{1}{\varphi_\infty}} dx\,\frac{x^{L-1}}{1-x} - \ln\left(1+\frac{1}{\varphi_\infty}\right)\right], \tag{A.3b}$$

$$H = \frac{\rho_\infty}{m}\frac{\left(\frac{\partial x_f}{\partial t} - v_1 + v_2\frac{\partial x_f}{\partial y} + v_3\frac{\partial x_f}{\partial z}\right)}{\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]^{1/2}}, \tag{A.3c}$$

$$\frac{1}{\delta_1} = \frac{\frac{\partial x_f}{\partial t} - v_1 + v_2\frac{\partial x_f}{\partial y} + v_3\frac{\partial x_f}{\partial z}}{\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]} \; . \tag{A.3d}$$

The variable $\delta_1(y,z,t)$ is a measure of the flame thickness viewed in a direction parallel to the x-axis in the sense that in the preheat zone the temperature has the form

$$\varphi \sim \varphi_\infty + \exp\left[-\frac{1}{\delta_1}\frac{m^2 C_p}{\lambda\rho_\infty}(x - x_f)\right] \; . \tag{A.4}$$

Our discussion is concerned with the physical interpretation of these equations, and their relation to flame stretch and other kinematic concepts.

The Kinematics of a Flame Sheet. We now examine the flame on the hydrodynamic scale and calculate the nondimensional stretch factor

$$S \equiv \frac{\lambda\rho_\infty}{m^2 C_p} \frac{1}{\Delta} \frac{d\Delta}{dt} \tag{A.5}$$

in terms of the arbitrary functions x_f, $\vec{v}$. The ultimate goal is to interpret the system (A.3) in terms of kinematic variables such as S, so that the discussion is carried out in a Cartesian frame.

The most transparent way to calculate S, though certainly not the most elegant, is to consider the deformation of an infinitesimal parallelogram as it moves over the flame surface. To this end, consider a point A attached to the surface with coordinates (x,y,z) at time t so that

$$x = x_f(y,z,t) \ .$$

Consider also a nearby point B with coordinates $\left(x_f + \epsilon \frac{\partial x_f}{\partial y},\ y + \epsilon, z\right)$ and a point C with coordinates $\left(x_f + \epsilon \frac{\partial x_f}{\partial z},\ y, z + \epsilon\right)$. These three points define an infinitesimal parallelogram in the flame sheet of area Δ where

$$\Delta = \epsilon^2 \left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]^{\frac{1}{2}} . \tag{A.6}$$

The velocity of A is labelled $\vec{q}(y,z,t) \equiv (q_1, q_2, q_3)$ and the velocities of B and C are respectively $\vec{q} + \epsilon \frac{\partial \vec{q}}{\partial y}$ and $\vec{q} + \epsilon \frac{\partial \vec{q}}{\partial z}$. Thus after a time δt the coordinates of A are $(x_f + q_1 \delta t,\ y + q_2 \delta t,\ z + q_3 \delta t)$ and this only remains in the flame surface, as required, if

$$q_1 = \frac{\partial x_f}{\partial t} + q_2 \frac{\partial x_f}{\partial y} + q_3 \frac{\partial x_f}{\partial z} . \tag{A.7}$$

The new coordinates of B are $\left(x_f + \epsilon \frac{\partial x_f}{\partial y} + q_1 \delta t + \epsilon \frac{\partial q_1}{\partial y} \delta t,\right.$

$\left. y + \epsilon + q_2 \delta t + \epsilon \frac{\partial q_2}{\partial y} \delta t,\ z + q_3 \delta t + \epsilon \frac{\partial q_3}{\partial y} \delta t\right)$ with a similar result for C

and in this way the new area of the parallelogram can be calculated, and after equating to $\Delta + \frac{d\Delta}{dt}\delta t$ yields the result

$$\frac{1}{\Delta}\frac{d\Delta}{dt} = \frac{\left[\frac{\partial q_2}{\partial y} + \frac{\partial q_3}{\partial z} - \frac{\partial x_f}{\partial y}\frac{\partial x_f}{\partial z}\frac{\partial q_3}{\partial y} - \frac{\partial x_f}{\partial y}\frac{\partial x_f}{\partial z}\frac{\partial q_2}{\partial z} + \left(\frac{\partial x_f}{\partial y}\right)^2\frac{\partial q_3}{\partial z} + \left(\frac{\partial x_f}{\partial z}\right)^2\frac{\partial q_2}{\partial y} + \frac{\partial x_f}{\partial y}\frac{\partial q_1}{\partial y} + \frac{\partial x_f}{\partial z}\frac{\partial q_1}{\partial z}\right]}{\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]}$$

or, eliminating q_1 using (A.7)

$$\frac{1}{\Delta}\frac{d\Delta}{dt} = \left(\frac{\partial q_2}{\partial y} + \frac{\partial q_3}{\partial z}\right) + \frac{1}{2}\frac{D}{Dt}\ln\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right], \tag{A.8}$$

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + q_2\frac{\partial}{\partial y} + q_3\frac{\partial}{\partial z} .$$

The velocity of the point A tangential to the sheet is equal to the tangential component of the gas velocity $\vec{v}$ so that

$$\vec{q} - (\vec{q}\cdot\vec{n})\vec{n} = \vec{v} - (\vec{v}\cdot\vec{n})\vec{n} \tag{A.9}$$

where

$$\vec{n} = \frac{\left(1, -\frac{\partial x_f}{\partial y}, -\frac{\partial x_f}{\partial z}\right)}{\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]^{\frac{1}{2}}} \tag{A.10}$$

is the unit normal to the flame surface. Noting that

$$(\vec{q}\cdot\vec{n}) = \frac{\frac{\partial x_f}{\partial t}}{\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]^{\frac{1}{2}}}, \tag{A.11}$$

equation (A.9) can be used to express $\vec{q}$ in terms of $\vec{v}$. Thus

$$\frac{1}{\Delta}\frac{d\Delta}{dt} = \left(\frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}\right) - \frac{\partial}{\partial y}\left(\frac{1}{\delta_1}\frac{\partial x_f}{\partial y}\right) - \frac{\partial}{\partial z}\left(\frac{1}{\delta_1}\frac{\partial x_f}{\partial z}\right)$$

$$+ \frac{1}{2}\frac{D}{Dt}\ln\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right] \tag{A.12}$$

where the operator $\frac{D}{Dt}$ can now be written in the form

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_2 \frac{\partial}{\partial y} + v_3 \frac{\partial}{\partial z} - \frac{1}{\delta_1} \frac{\partial x_f}{\partial y} \frac{\partial}{\partial y} - \frac{1}{\delta_1} \frac{\partial x_f}{\partial z} \frac{\partial}{\partial z} \; . \tag{A.13}$$

Note that if v_2 and v_3 both vanish identically and v_1, x_f depend only on y, the stretch factor may be written as

$$\frac{1}{\Delta}\frac{d\Delta}{dt} = \frac{dv_1}{dy} \cdot \frac{x_f'}{1 + (x_f')^2} + \frac{v_1 x_f''}{(1+(x_f')^2)^2} \tag{A.14}$$

which differs significantly from the Karlovitz Number (equation 3.3).

Consider now the variations in the thickness $\delta(y,z,t)$ of the element of the flame as it moves over the surface. In view of (A.10),

$$\delta = \frac{\delta_1}{\left[1 + \left(\frac{\partial x_f}{\partial y}\right)^2 + \left(\frac{\partial x_f}{\partial z}\right)^2\right]^{\frac{1}{2}}} \tag{A.15}$$

and moreover

$$\frac{d\delta}{dt} = \frac{D\delta}{Dt} \, . \tag{A.16}$$

The volume V of the flame element, defined by

$$V = \delta\Delta$$

changes according to

$$\frac{1}{V}\frac{dV}{dt} = \frac{1}{\Delta}\frac{d\Delta}{dt} + \frac{1}{\delta}\frac{d\delta}{dt}$$

so that from (A.12) and (A.10) it follows that

$$\frac{1}{V}\frac{dV}{dt} = \frac{1}{\delta_1}\left(\frac{\partial \delta_1}{\partial t} + v_2 \frac{\partial \delta_1}{\partial y} + v_3 \frac{\partial \delta_1}{\partial z}\right) + \left(\frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}\right) - \frac{1}{\delta_1}\left(\frac{\partial^2 x_f}{\partial y^2} + \frac{\partial^2 x_f}{\partial z^2}\right) \; . \tag{A.17}$$

Finally we note that the flame speed is defined as the normal component of the flame element velocity relative to the gas, i.e.

$$\text{Flame Speed} = (\vec{q}\cdot\vec{n}) - (\vec{v}\cdot\vec{n}) = \frac{\dfrac{\partial x_f}{\partial t} - v_1 + v_2\dfrac{\partial x_f}{\partial y} + v_3\dfrac{\partial x_f}{\partial z}}{\left[1 + \left(\dfrac{\partial x_f}{\partial y}\right)^2 + \left(\dfrac{\partial x_f}{\partial z}\right)^2\right]^{\frac{1}{2}}} \tag{A.18}$$

and the thickness δ is inversely proportional to this quantity.

Interpretation of the Activation Energy Results. Comparison of the asymptotic results (A.3) and the kinematic expression (A.17) shows that the fundamental equation governing the propagation of a slowly varying flame can be written in the form

$$H^2 \ell n(H^2) - \beta\,\frac{1}{V}\frac{dV}{dt} = 0 \tag{A.19}$$

where H may be identified with the nondimensional flame speed. Note that the effect of a change in volume depends on the value of the Lewis Number L. If $L > 1$ then β is negative so that dilatation $\left(\frac{dV}{dt} > 0\right)$ slows the flame below the adiabatic value $(H^2 < 1)$ whereas compression $\left(\frac{dV}{dt} < 0\right)$ speeds it up $(H^2 > 1)$. If the Lewis Number is less than one this effect is reversed.

Since $\delta H = \rho_\infty / m$, (A.19) can be written in the form

$$H^2 \ell n(H^2) + \frac{\beta}{H}\frac{dH}{dt} = \frac{\beta}{\Delta}\frac{d\Delta}{dt}\,, \tag{A.20}$$

thus establishing a functional relationship between H and S.

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An analysis is undertaken, both mathematical and numerical, of the quenching of two-dimensional premixed flames under a variety of circumstances. The discussion is divided into two parts, the first of which deals with quenching due to proximity to a surface through which there are heat losses, the second, quenching due to a shear flow of the kind experienced by a flame attached to a wire. The discussion includes a critical appraisal of flame stretch and the role sometimes claimed for it in intuitive explanations of flame quenching.

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